CHAPTER

2

Rational Numbers

When you think of your favourite game, what comes to mind? It may be a computer game or video game. You may also enjoy playing games that have been around a lot longer. These may include the use of a game board and may involve cards, dice, or specially designed playing pieces. Examples of these games include chess, checkers, dominoes, euchre, bridge, Monopoly™, and Scrabble®.

In this chapter, you will learn more about games and about how you can use rational numbers to describe or play them. You will also design your own game.

Did You Know?

Canadians have invented many popular board games, such as crokinole, Yahtzee®, Trivial Pursuit®, Balderdash™, and Scruples™.

WWW Web Link

For more information about board games invented by Canadians, go to www.mathlinks9.ca and follow the links.

What You Will Learn

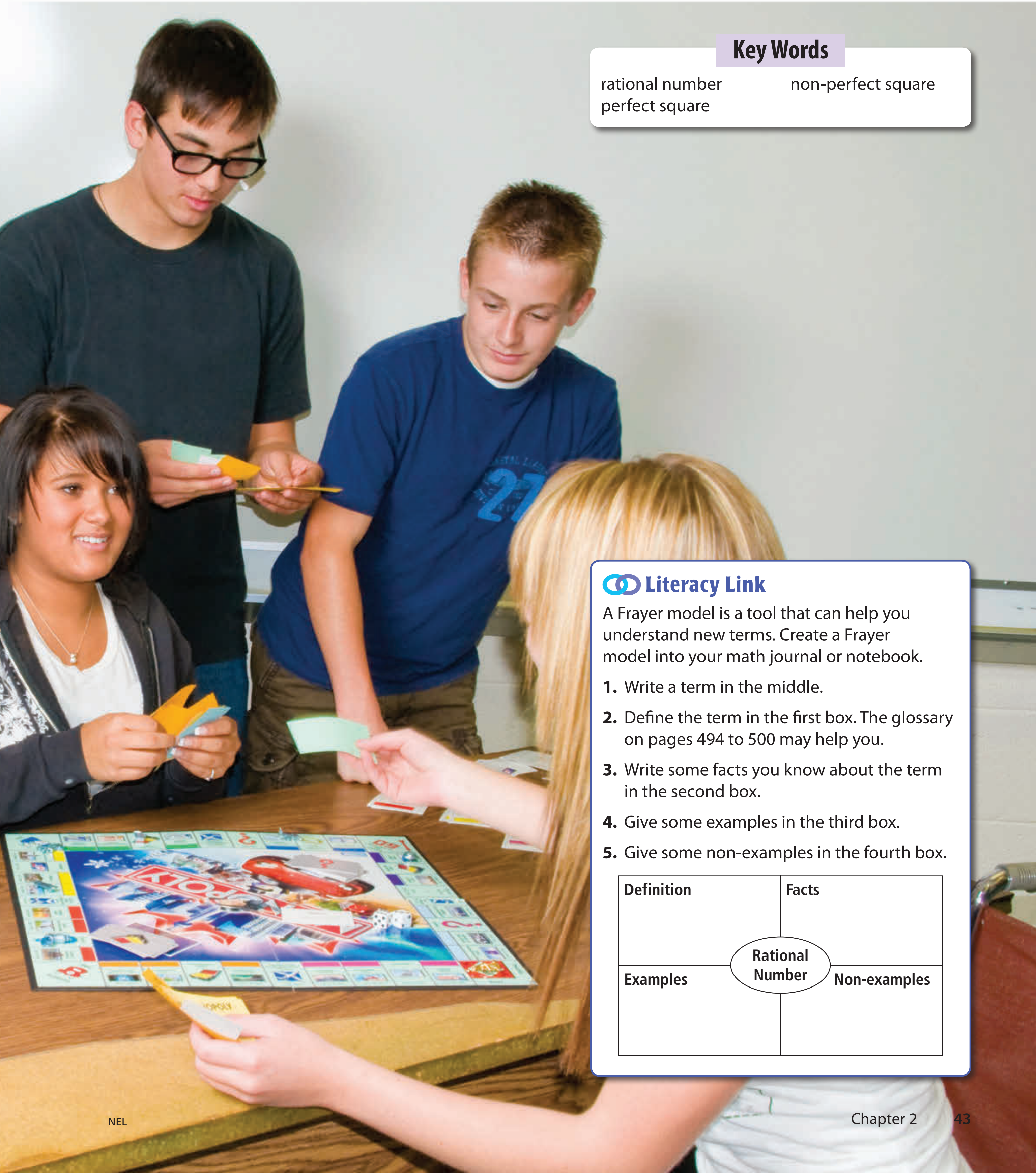
- to compare and order rational numbers
- to solve problems involving operations on rational numbers
- to determine the square root of a perfect square rational number
- to determine the approximate square root of a non-perfect square rational number

Key Words

rational number

perfect square

non-perfect square



Literacy Link

A Frayer model is a tool that can help you understand new terms. Create a Frayer model into your math journal or notebook.

1. Write a term in the middle.
2. Define the term in the first box. The glossary on pages 494 to 500 may help you.
3. Write some facts you know about the term in the second box.
4. Give some examples in the third box.
5. Give some non-examples in the fourth box.

Definition	Facts
Examples	Non-examples

Rational Number

FOLDABLES™
Study Tool

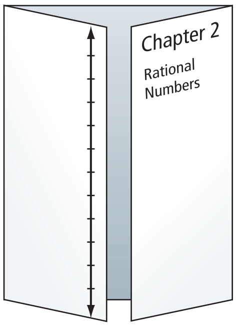
Making the Foldable

Materials

- sheet of 11 × 17 paper
- three sheets of 8.5 × 11 paper
- sheet of grid paper
- ruler
- scissors
- stapler

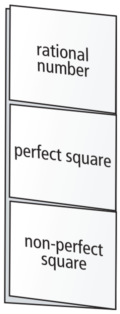
Step 1

Fold the long side of a sheet of 11 × 17 paper in half. Pinch it at the midpoint. Fold the outer edges of the paper to meet at the midpoint. Write the chapter title and draw a number line as shown.



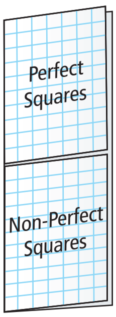
Step 2

Fold the short side of a sheet of 8.5 × 11 paper in half. Fold in three the opposite way. Make two cuts as shown through one thickness of paper, forming three tabs. Label the tabs as shown.



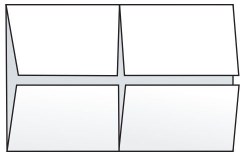
Step 3

Fold the short side of a sheet of 8.5 × 11 of grid paper in half. Fold in half the opposite way. Make a cut through one thickness of paper, forming two tabs. Label the tabs as shown.



Step 4

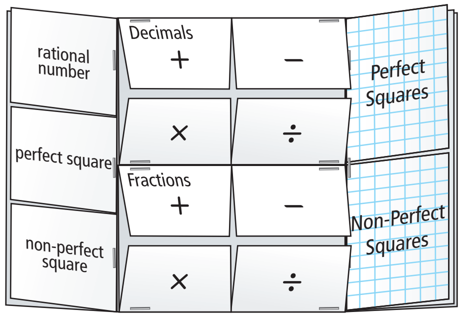
Fold the long side of a sheet of 8.5 × 11 paper in half. Pinch it at the midpoint. Fold the outer edges of the paper to meet at the midpoint. Fold the long side of the folded paper in half. Cut as shown, forming four doors.



Repeat Step 4 to make another four-door book. Label the doors as shown below.

Step 5

Staple the four booklets you made into the Foldable from Step 1 as shown.



Using the Foldable

As you work through the chapter, write the definitions of the Key Words beneath the tabs on the left. Beneath the tabs on the right, define and show examples of square roots of perfect squares and non-perfect squares. Beneath the centre tabs, provide examples of adding, subtracting, multiplying, and dividing rational numbers in decimal form and fraction form.

On the back of the Foldable, make notes under the heading What I Need to Work On. Check off each item as you deal with it.

Math Link

Problem Solving With Games

Millions of Canadians enjoy the challenge and fun of playing chess. Early versions of this game existed in India over 1400 years ago. The modern version of chess emerged from southern Europe over 500 years ago.

1. If each of the small squares on a chessboard has a side length of 3 cm, what is the total area of the dark squares? Solve this problem in two ways.
2. If the total area of a chessboard is 1024 cm^2, what is the side length of each of the smallest squares?
3. For the chessboard in #2, what is the length of a diagonal of the board? Express your answer to the nearest tenth of a centimetre.
4. Compare your solutions with your classmates' solutions.

In this chapter, you will describe or play other games by solving problems that involve decimals, fractions, squares, and square roots. You will then use your skills to design a game of your own.

2.1 Comparing and Ordering Rational Numbers

Focus on...

After this lesson, you will be able to...

- compare and order rational numbers
- identify a rational number between two given rational numbers

Cape Dorset, Nunavut

The percent of Canadians who live in rural areas has been decreasing since 1867. At that time, about 80% of Canadians lived in rural areas. Today, about 80% of Canadians live in urban areas, mostly in cities. The table shows changes in the percent of Canadians living in urban and rural areas over four decades.

Did You Know?

An urban area has a population of 1000 or more. In urban areas, 400 or more people live in each square kilometre. Areas that are not urban are called rural. What type of area do you live in?

Decade	Change in the Percent of Canadians in Urban Areas (%)	Change in the Percent of Canadians in Rural Areas (%)
1966–1976	+1.9	−1.9
1976–1986	+1.0	−1.0
1986–1996	+1.4	−1.4
1996–2006	+2.3	−2.3

How can you tell that some changes in the table are increases and others are decreases?

Explore Rational Numbers

1. How are the rational numbers in the table on page 46 related. Explain your reasoning.

2. a) Choose a rational number in decimal form. Identify its opposite. How do you know these are opposite rational numbers?

b) Choose a rational number in fraction form. Identify its opposite.

c) Identify another pair of opposite rational numbers.

3. a) Identify equivalent rational numbers from the following list.

$\frac{12}{4}$ $\frac{-8}{4}$ $\frac{-9}{-3}$ $-\frac{4}{2}$ $\frac{4}{-2}$ $\frac{12}{3}$ $-\left(\frac{4}{-1}\right)$ $-\left(\frac{-4}{-2}\right)$

b) Choose a rational number in fraction form that is not equivalent to any of the rational numbers in part a). Challenge a classmate to write four rational numbers that are equivalent to your chosen number.

Reflect and Check

4. a) How can you identify opposite rational numbers?

b) How can you identify equivalent rational numbers?

5. a) Predict what you think the change in the percent of Canadians in urban areas from 2006 to 2016 might be. Justify your prediction.

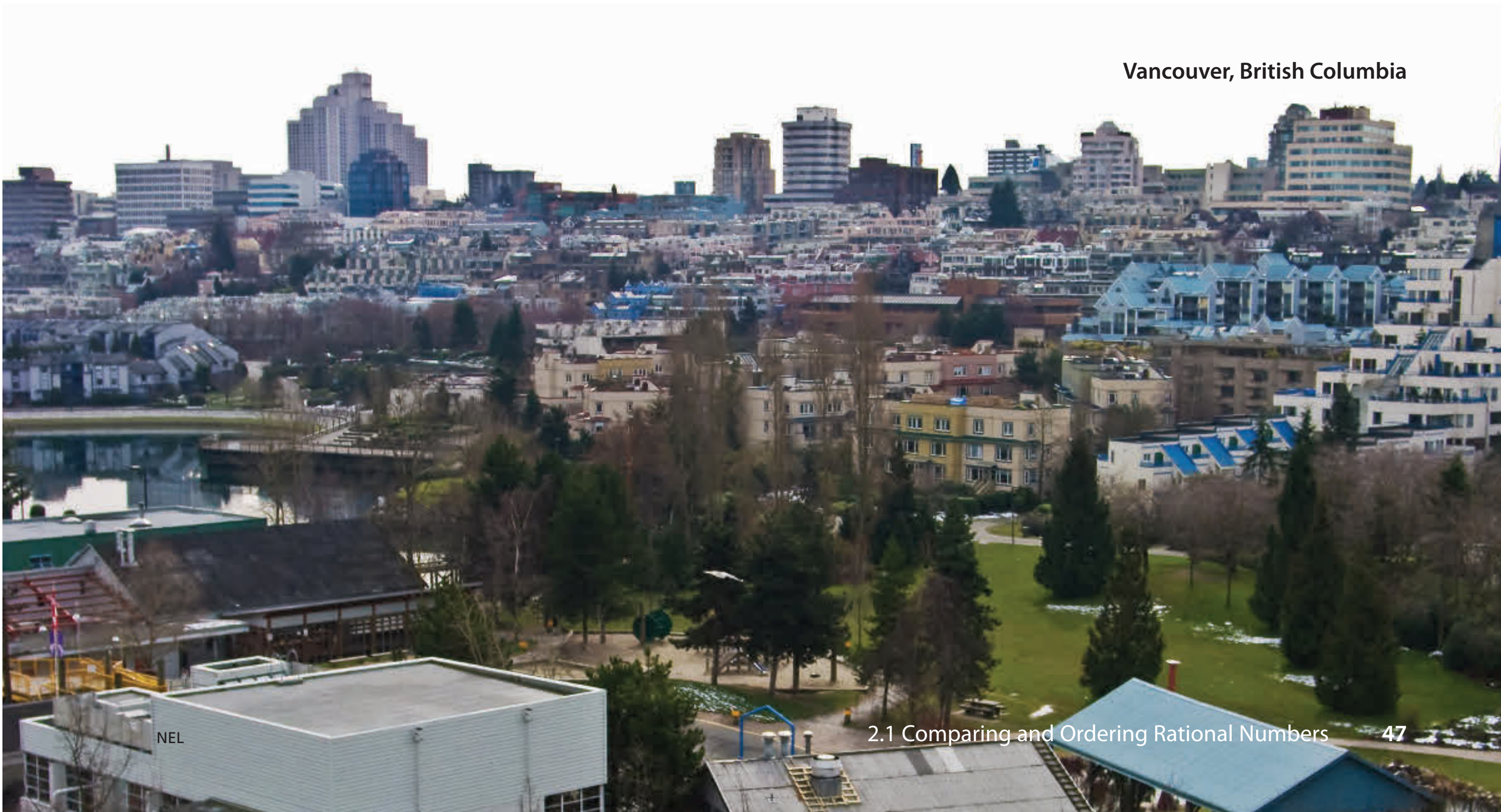
b) What would you expect the change in the percent of Canadians in rural areas to be for that decade? Explain.

rational number

- a number that can be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$
- examples include −4, 3.5, $-\frac{1}{2}$, $1\frac{3}{4}$, and 0

Literacy Link

When numbers are *equivalent*, they have the same value. $\frac{24}{-4}$, $\frac{-18}{3}$, $-\frac{12}{2}$, and $-\left(\frac{-6}{-1}\right)$ are all equivalent. They all represent the same rational number. What is it?



Vancouver, British Columbia

Link the Ideas

Example 1: Compare and Order Rational Numbers

Compare and order the following rational numbers.

$-1.2 \qquad \frac{4}{5} \qquad \frac{7}{8} \qquad -0.\overline{5} \qquad -\frac{7}{8}$

Solution

You can estimate the order.

-1.2 is a little less than -1.

$\frac{4}{5}$ is a little less than 1.

$\frac{7}{8}$ is a little less than 1.

$-0.\overline{5}$ is a little less than -0.5.

$-\frac{7}{8}$ is a little more than -1.

An estimate of the order from least to greatest is $-1.2, -\frac{7}{8}, -0.\overline{5}, \frac{4}{5}, \frac{7}{8}$.

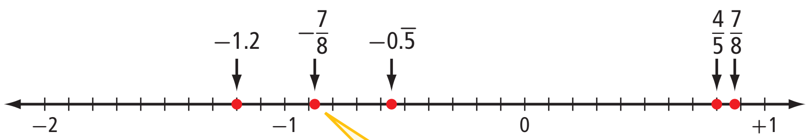
Strategies

Draw a Diagram

Express all the numbers in the same form.
You can write the numbers in decimal form.

$-1.2 \qquad \frac{4}{5} = 0.8 \qquad \frac{7}{8} = 0.875 \qquad -0.\overline{5} = -0.555... \qquad -\frac{7}{8} = -0.875$

Place the numbers on a number line.



What number is the opposite of $-\frac{7}{8}$? How does the position of that number on the number line compare with the position of $-\frac{7}{8}$?

The numbers in ascending order are $-1.2, -\frac{7}{8}, -0.\overline{5}, \frac{4}{5}$, and $\frac{7}{8}$.

The numbers in descending order are $\frac{7}{8}, \frac{4}{5}, -0.\overline{5}, -\frac{7}{8}$, and -1.2.

Show You Know

Compare the following rational numbers. Write them in ascending order and descending order.

$0.\overline{3} \qquad -0.6 \qquad -\frac{3}{4} \qquad 1\frac{1}{5} \qquad -1$

Example 2: Compare Rational Numbers

Which fraction is greater, $-\frac{3}{4}$ or $-\frac{2}{3}$?

Solution

Method 1: Use Equivalent Fractions

You can express the fractions as equivalent fractions with a common denominator.

A common denominator of the two fractions is 12.

> How do you know 12 is a common denominator?

$$-\frac{3}{4} = -\frac{9}{12} \quad (\times 3) \qquad -\frac{2}{3} = -\frac{8}{12} \quad (\times 4)$$

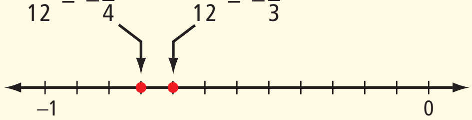
When the denominators are the same, compare the numerators.

$$-\frac{9}{12} = \frac{-9}{12} \qquad -\frac{8}{12} = \frac{-8}{12}$$

$\frac{-8}{12} > \frac{-9}{12}$, because $-8 > -9$.

$-\frac{2}{3}$ is the greater fraction.

> How does the number line show the comparison?
>
> $-\frac{9}{12} = -\frac{3}{4}$ $\qquad$ $-\frac{8}{12} = -\frac{2}{3}$



Method 2: Use Decimals

You can also compare by writing the fractions as decimal numbers.

$-\frac{3}{4} = -0.75$

$-\frac{2}{3} = -0.\overline{6}$

$-0.\overline{6} > -0.75$

$-\frac{2}{3}$ is the greater fraction.

Literacy Link

The quotient of two integers with unlike signs is negative. This means that

$-\frac{9}{12} = \frac{-9}{12} = \frac{9}{-12}$

and

$-\frac{8}{12} = \frac{-8}{12} = \frac{8}{-12}$.

Show You Know

Which fraction is smaller, $-\frac{7}{10}$ or $-\frac{3}{5}$?

WWW Web Link

For practice comparing and ordering rational numbers, go to www.mathlinks9.ca and follow the links.

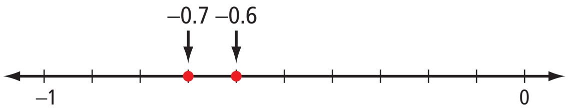
Example 3: Identify a Rational Number Between Two Given Rational Numbers

Identify a fraction between −0.6 and −0.7.

Strategies
Draw a Diagram

Solution

You can first identify a decimal number between −0.6 and −0.7, using a number line.



You can also change −0.6 and −0.7 into fraction form. What would the number line look like?

One decimal number between −0.6 and −0.7 is −0.65.

Convert the decimal to a fraction. $-0.65 = -\frac{65}{100}$

A fraction between −0.6 and −0.7 is $-\frac{65}{100}$.

What is another way to express $-\frac{65}{100}$ as a fraction?

Show You Know

Identify a fraction between −2.4 and −2.5.

Key Ideas

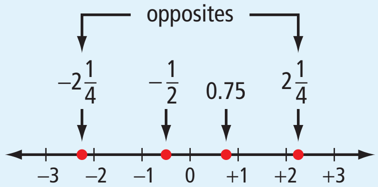
- Rational numbers can be positive, negative, or zero. They include integers, positive and negative fractions, mixed numbers, and decimal numbers.

  Examples: $-6, 15, \frac{3}{4}, -1\frac{2}{3}, 3.9, -2.\overline{3}$

- Equivalent fractions represent the same rational number.

  $-\frac{5}{2}, \frac{-5}{2}, \frac{10}{-4}$, and $-\left(\frac{-10}{-4}\right)$ all represent $-2\frac{1}{2}$ or −2.5.

- One strategy for comparing and ordering rational numbers is to use a number line.
  - On a horizontal number line, a larger rational number is to the right of a smaller rational number.
  - Opposite rational numbers are the same distance in opposite directions from zero.

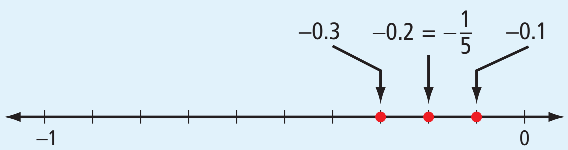


- You can compare fractions with the same denominator by comparing the numerators.

  $\frac{-7}{10} < \frac{-6}{10}$, because $-7 < -6$.

- One strategy for identifying a rational number between two given rational numbers is to use a number line.

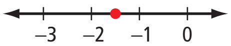
  A rational number in fraction form between −0.3 and −0.1 is $\frac{-1}{5}$.



Check Your Understanding

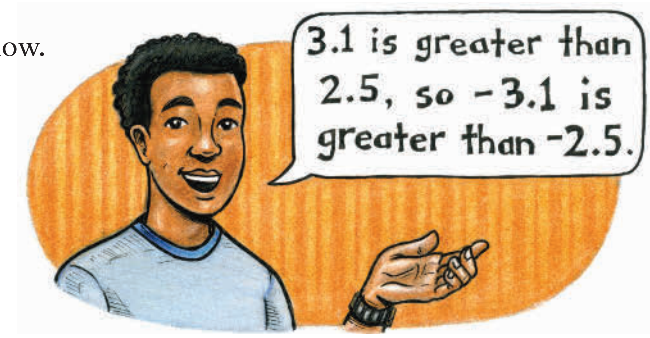
Communicate the Ideas

1. Laura placed $-2\frac{1}{2}$ incorrectly on a number line, as shown.



How could you use the idea of opposites to show Laura how to plot $-2\frac{1}{2}$ correctly?

2. Is Dominic correct? Show how you know.



3. Tomas and Roxanne were comparing -0.9 and $-\frac{7}{8}$. Tomas wrote -0.9 as a fraction, and then he compared the two fractions. Roxanne wrote $-\frac{7}{8}$ as a decimal, and then she compared the two decimals.

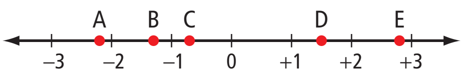
a) Which method do you prefer? Explain.

b) Which is greater, -0.9 or $-\frac{7}{8}$? Explain how you know.

Practise

For help with #4 to #9, refer to Example 1 on page 48.

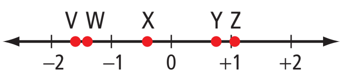
4. Match each rational number to a point on the number line.



a) $\frac{3}{2}$ **b)** -0.7 **c)** $-2\frac{1}{5}$

d) $\frac{14}{5}$ **e)** $-1\frac{1}{3}$

5. Which point on the number line matches each rational number?



a) $-1\frac{2}{5}$ **b)** $\frac{3}{4}$ **c)** $1\frac{1}{20}$

d) $-1\frac{3}{5}$ **e)** $-0.\overline{4}$

6. Place each number and its opposite on a number line.

a) $\frac{8}{9}$ **b)** -1.2 **c)** $2\frac{1}{10}$ **d)** $-\frac{11}{3}$

7. What is the opposite of each rational number?

a) $-4.\overline{1}$ **b)** $\frac{4}{5}$ **c)** $-5\frac{3}{4}$ **d)** $\frac{9}{8}$

8. Compare $1\frac{5}{6}$, $-1\frac{2}{3}$, -0.1, 1.9, and $-\frac{1}{5}$. Write the numbers in ascending order.

9. Compare $-\frac{3}{8}$, $1.\overline{8}$, $\frac{9}{5}$, $-\frac{1}{2}$, and -1. Write the numbers in descending order.

For help with #10 to #13, refer to Example 2 on page 49.

10. Express each fraction as an equivalent fraction.

a) $-\frac{2}{5}$ **b)** $\frac{10}{6}$

c) $-\frac{9}{12}$ **d)** $\frac{-4}{3}$

11. Write each rational number as an equivalent fraction.

a) $\frac{-1}{3}$ **b)** $\frac{-4}{-5}$

c) $-\left(\frac{-5}{-4}\right)$ **d)** $\frac{7}{-2}$

12. Which value in each pair is greater?

a) $\frac{1}{3}, -\frac{2}{3}$ **b)** $-\frac{9}{10}, \frac{7}{10}$

c) $-\frac{1}{2}, -\frac{3}{5}$ **d)** $-2\frac{1}{8}, -2\frac{1}{4}$

13. Which value in each pair is smaller?

a) $\frac{4}{7}, \frac{2}{3}$ **b)** $-\frac{4}{3}, -\frac{5}{3}$

c) $-\frac{7}{10}, -\frac{3}{5}$ **d)** $-1\frac{3}{4}, -1\frac{4}{5}$

For help with #14 to #17, refer to Example 3 on page 50.

14. Identify a decimal number between each of the following pairs of rational numbers.

a) $\frac{3}{5}, \frac{4}{5}$ **b)** $-\frac{1}{2}, -\frac{5}{8}$

c) $-\frac{5}{6}, 1$ **d)** $-\frac{17}{20}, -\frac{4}{5}$

15. What is a decimal number between each of the following pairs of rational numbers?

a) $1\frac{1}{2}, 1\frac{7}{10}$ **b)** $-2\frac{2}{3}, -2\frac{1}{3}$

c) $1\frac{3}{5}, -1\frac{7}{10}$ **d)** $-3\frac{1}{100}, -3\frac{1}{50}$

16. Identify a fraction between each of the following pairs of rational numbers.

a) 0.2, 0.3 **b)** 0, −0.1

c) −0.74, −0.76 **d)** −0.52, −0.53

17. Identify a mixed number between each of the following pairs of rational numbers.

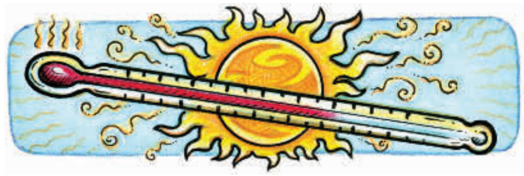
a) 1.7, 1.9 **b)** −0.5, 1.5

c) −3.3, −3.4 **d)** −2.01, −2.03

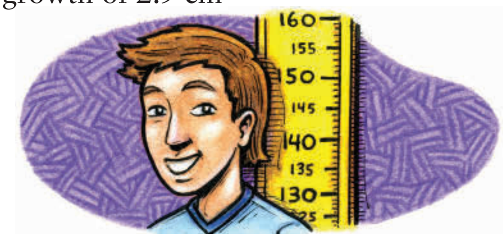
Apply

18. Use a rational number to represent each quantity. Explain your reasoning.

a) a temperature increase of 8.2 °C



b) growth of 2.9 cm



c) 3.5 m below sea level

d) earnings of $32.50

e) 14.2 °C below freezing

19. The table includes the melting points and boiling points of six elements known as the noble gases.

Noble Gas	Melting Point (°C)	Boiling Point (°C)
Argon	−189.2	−185.7
Helium	−272.2	−268.6
Neon	−248.67	−245.92
Krypton	−156.6	−152.3
Radon	−71.0	−61.8
Xenon	−111.9	−107.1

a) Which noble gases have a melting point that is less than the melting point of argon?

b) Which noble gases have a boiling point that is greater than the boiling point of krypton?

c) Arrange the melting points in ascending order.

d) Arrange the boiling points in descending order.

Science Link

For many years, the noble gases were known as the *inert gases*. Most chemists thought that these gases would not react with other chemicals. In 1962, Neil Bartlett, a chemist at the University of British Columbia, proved them wrong.

WWW Web Link

To learn more about Neil Bartlett and to research Canadian scientific discoveries, go to www.mathlinks9.ca and follow the links.

20. a) Kwasi said that he ignored the fractions when he decided that $-2\frac{1}{5}$ is smaller than $-1\frac{9}{10}$. Explain his thinking.

b) Naomi said that she ignored the integer −1 when she decided that $-1\frac{1}{4}$ is greater than $-1\frac{2}{7}$. Explain her thinking.

21. The table shows the average early-morning temperature for seven communities in May.

Community	Average Early-Morning Temperature (°C)
Churchill, Manitoba	−5.1
Regina, Saskatchewan	3.9
Edmonton, Alberta	5.4
Penticton, British Columbia	6.1
Yellowknife, Northwest Territories	−0.1
Whitehorse, Yukon Territory	0.6
Resolute, Nunavut	−14.1

a) Write the temperatures in descending order.

b) Which community has an average temperature between the values for Whitehorse and Churchill?

22. Replace each ■ with >, <, or = to make each statement true.

a) $\frac{-9}{6}$ ■ $\frac{3}{-2}$

b) $-\frac{3}{5}$ ■ $-0.\overline{6}$

c) $-1\frac{3}{10}$ ■ $-\left(\frac{-13}{-10}\right)$

d) -3.25 ■ $-3\frac{1}{5}$

e) $-\frac{8}{12}$ ■ $-\frac{11}{15}$

f) $-2\frac{5}{6}$ ■ $-2\frac{7}{8}$

23. Is zero a rational number? Explain.

24. Give an example of a fraction in lowest terms that satisfies the following conditions.

a) greater than 0, with the denominator greater than the numerator

b) between 0 and −1, with the denominator less than the numerator

c) less than −2, with the numerator less than the denominator

d) between −1.2 and −1.3, with the numerator greater than the denominator

25. Which integers are between $\frac{11}{5}$ and $\frac{15}{-4}$?

26. Which number in each pair is greater? Explain each answer.

a) 0.4 and 0.44

b) $0.\overline{3}$ and 0.33

c) −0.7 and −0.77

d) −0.66 and $-0.\overline{6}$

27. Identify the fractions that are between 0 and −2 and that have 3 as the denominator.

Extend

28. How many rational numbers are between $\frac{2}{3}$ and $0.\overline{6}$? Explain.

29. Replace each $\blacksquare$ with an integer to make each statement true. In each case, is more than one answer possible? Explain.

a) $\blacksquare.5 < -1.9$

b) $\frac{\blacksquare}{-4} = -2\frac{1}{4}$

c) $\frac{-3}{\blacksquare} = -\frac{-15}{5}$

d) $-1.5\blacksquare2 > -1.512$

e) $-\frac{3}{4} < -0.7\blacksquare$

f) $-5\frac{1}{2} > \frac{11}{\blacksquare}$

g) $-2\frac{3}{5} = \frac{\blacksquare}{10}$

h) $\frac{8}{\blacksquare} < -\frac{2}{3}$

30. Determine the value of x.

a) $\frac{4}{-5} = \frac{x}{-10}$

b) $\frac{x}{3} = \frac{6}{-9}$

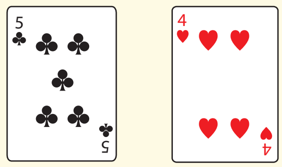
c) $\frac{5}{x} = -\frac{20}{12}$

d) $\frac{-6}{-5} = \frac{30}{x}$

Math Link

Play the following game with a partner or in a small group. You will need one deck of playing cards.

- Remove the jokers, aces, and face cards from the deck.
- Red cards represent positive integers. Black cards represent negative integers.
- In each round, the dealer shuffles the cards and deals two cards to each player.
- Use your two cards to make a fraction that is as close as possible to zero.
- In each round, the player with the fraction closest to zero wins two points. If there is a tie, each tied player wins a point.
- The winner is the first player with ten points. If two or more players reach ten points in the same round, keep playing until one player is in the lead by at least two points.



represents −5   represents 4

With a five of clubs and a four of hearts, you can make $\frac{4}{-5}$ or $\frac{-5}{4}$. Choose $\frac{4}{-5}$ because it is closer to zero.

2.2 Problem Solving With Rational Numbers in Decimal Form

Focus on…

After this lesson, you will be able to…

- **perform operations on rational numbers in decimal form**
- **solve problems involving rational numbers in decimal form**

Did You Know?

As Canada's sunniest provincial capital, Regina averages almost 6.5 h of sunshine per day. That is over 2 h per day more sunshine than St. John's, Newfoundland and Labrador. St. John's is the least sunny provincial capital.

In Regina, Saskatchewan, the average mid-afternoon temperature in January is −12.6 °C. The average mid-afternoon temperature in July is 26.1 °C. Estimate how much colder Regina is in January than in July.

Explore Multiplying and Dividing Rational Numbers in Decimal Form

1. a) Estimate the products and quotients. Explain your method.

3.2×4.5 $\quad 3.2 \times (-4.5)$ $\quad -20.9 \div 9.5$ $\quad -20.9 \div (-9.5)$

b) Calculate the products and quotients in part a). Explain your method.

2. a) Suppose the temperature one January afternoon in Regina decreased by 2.6 °C every hour for 3.5 h. What was the overall temperature change during that time?

b) Suppose the temperature in Regina one July afternoon increased by 9.9 °C in 5.5 h. What was the average temperature change per hour?

Reflect and Check

3. How can you multiply and divide rational numbers in decimal form?

4. Create a problem that can be solved using the multiplication or division of rational numbers. Challenge a classmate to solve it.

Link the Ideas

Example 1: Add and Subtract Rational Numbers in Decimal Form

Estimate and calculate.

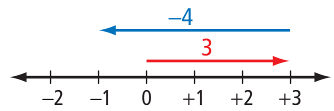
a) $2.65 + (-3.81)$
b) $-5.96 - (-6.83)$

Solution

a) Estimate.

$$2.65 + (-3.81)$$
$$\approx 3 + (-4)$$
$$\approx -1$$

How does the number line show the estimate?



Method 1: Use Paper and Pencil

Calculate.
Adding the opposite of 3.81 is the same as subtracting 3.81.
$2.65 + (-3.81) = 2.65 - 3.81$

Determine the difference between 3.81 and 2.65.
$3.81 - 2.65 = 1.16$
However, $2.65 - 3.81$ must be negative, since $3.81 > 2.65$.
So, $2.65 + (-3.81) = -1.16$.

Method 2: Use a Calculator

Calculate. C 2.65 + 3.81 +⇄– = –1.16

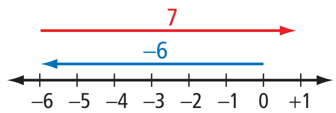
Tech Link

The key for entering a negative sign may look different on different calculators, for example, +⇄–, +/– or (–). It is not the subtraction key, –. Experiment with calculations involving negative signs on your calculator.

b) Estimate.

$$-5.96 - (-6.83)$$
$$\approx -6 - (-7)$$
$$\approx -6 + 7$$
$$\approx 1$$

How does the number line show the estimate?



Method 1: Use Paper and Pencil

Calculate.
$-5.96 - (-6.83) = -5.96 + 6.83$

Why is subtracting –6.83 the same as adding its opposite, 6.83?

Determine $6.83 + (-5.96)$.

$$6.83 + (-5.96)$$
$$= 6.83 - 5.96$$
$$= 0.87$$

So, $-5.96 - (-6.83) = 0.87$.

Why is –5.96 + 6.83 the same as 6.83 + (–5.96)?

Is the calculated answer reasonable? How do you know?

Method 2: Use a Calculator

Calculate. C 5.96 +⇄– – 6.83 +⇄– = 0.87

Show You Know

Estimate and calculate.

a) $-4.38 + 1.52$

b) $-1.25 - 3.55$

Example 2: Multiply and Divide Rational Numbers in Decimal Form

Estimate and calculate.

a) $0.45 \times (-1.2)$

b) $-2.3 \div (-0.25)$

Solution

a) Estimate.

$0.5 \times (-1) = -0.5$

Calculate.

Method 1: Use Paper and Pencil

You can calculate by multiplying the decimal numbers.

$0.45 \times 1.2 = 0.54$

Determine the sign of the product.

$0.45 \times (-1.2) = -0.54$

How do you know what the sign of the product is?

Method 2: Use a Calculator

C 0.45 × 1.2 +⇄− = −0.54

b) Estimate.

$-2.3 \div (-0.25)$

$\approx -2 \div (-0.2)$

≈ 10

Calculate.

C 2.3 +⇄− ÷ 0.25 +⇄− = 9.2

Show You Know

Estimate and calculate.

a) $-1.4(-2.6)$

b) $-2.76 \div 4.6$

Literacy Link

Parentheses is another name for brackets. They can be used in place of a multiplication sign. For example,

$-4 \times 1.5 = -4(1.5)$

Example 3: Apply Operations With Rational Numbers in Decimal Form

On Saturday, the temperature at the Blood Reserve near Stand Off, Alberta decreased by 1.2 °C/h for 3.5 h. It then decreased by 0.9 °C/h for 1.5 h.

a) What was the total decrease in temperature?

b) What was the average rate of decrease in temperature?

Why are the time periods represented by positive rational numbers? Why are the rates of temperature decrease represented by negative rational numbers?

Solution

a) The time periods can be represented by 3.5 and 1.5. The rates of temperature decrease can be represented by −1.2 and −0.9.

Method 1: Calculate in Stages

You can represent the temperature decrease in the first 3.5 h by $3.5 \times (-1.2) = -4.2$.

You can represent the temperature decrease in the last 1.5 h by $1.5 \times (-0.9) = -1.35$.

Add to determine the total temperature decrease.

$-4.2 + (-1.35) = -5.55$

The total decrease in temperature was 5.55 °C.

ME:
$4 \times (-1) = -4$
$1.5 \times (-1) = -1.5$
$-4 + (-1.5) = -5.5$

Method 2: Evaluate One Expression

The total temperature decrease can be represented by $3.5 \times (-1.2) + 1.5 \times (-0.9)$.

ME: $4 \times (-1) + 1.5 \times (-1) = -5.5$

Evaluate this expression, using the order of operations.

$$\begin{aligned} & 3.5 \times (-1.2) + 1.5 \times (-0.9) \\ &= -4.2 + (-1.35) \\ &= -5.55 \end{aligned}$$

You can also use a calculator.

C 3.5 × 1.2 +/− + 1.5 × 0.9 +/− = -5.55

The total decrease in temperature was 5.55 °C.

Literacy Link

Order of Operations

- Perform operations inside parentheses first.
- Multiply and divide in order from left to right.
- Add and subtract in order from left to right.

b) The average rate of decrease in temperature is the total decrease divided by the total number of hours.

The total number of hours is $3.5 + 1.5 = 5$.

$\frac{-5.55}{5} = -1.11$

ME: $-5 \div 5 = -1$

The average rate of decrease in temperature was 1.11 °C/h.

Show You Know

A hot-air balloon climbed at 0.8 m/s for 10 s.
It then descended at 0.6 m/s for 6 s.

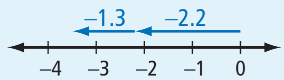
a) What was the overall change in altitude?

b) What was the average rate of change in altitude?

Key Ideas

- One way to model the addition of rational numbers is on a number line. One way to model the subtraction of rational numbers is by adding the opposite on a number line.

  $-2.2 - 1.3 = -3.5$ or $-2.2 + (-1.3) = -3.5$



- The product or quotient of two rational numbers with the same signs is positive.

  $-1.2 \times (-1.5) = 1.8$ $-1.5 \div (-1.2) = 1.25$

  The product or quotient of two rational numbers with different signs is negative.

  $1.2 \times (-1.5) = -1.8$ $-1.5 \div 1.2 = -1.25$

- The order of operations for calculations involving rational numbers is the same as for whole numbers, decimals, and integers.
  - Perform operations inside parentheses first.
  - Divide and multiply in order from left to right.
  - Add and subtract in order from left to right.

Check Your Understanding

Communicate the Ideas

1. a) Would you expect the subtraction $-3.5 - (-4.3)$ to give a positive answer or a negative answer? Explain.

b) Evaluate $-3.5 - (-4.3)$.

2. How do the values of the following two products compare? Explain your reasoning.

$2.54 \times (-4.22)$ -2.54×4.22

3. Leslie evaluated $-2.2 + 4.6 \times (-0.5)$ as -1.2. Zack evaluated the same expression as -4.5. Who was correct? Explain.

Practise

For help with #4 and #5, refer to Example 1 on page 56.

4. Estimate and calculate.

a) $0.98 + (-2.91)$

b) $5.46 - 3.16$

c) $-4.23 + (-5.75)$

d) $-1.49 - (-6.83)$

5. Calculate.

a) $9.37 - 11.62$ **b)** $-0.512 + 2.385$

c) $0.675 - (-0.061)$ **d)** $-10.95 + (-1.99)$

For help with #6 and #7, refer to Example 2 on page 57.

6. Estimate and calculate.

a) $2.7 \times (-3.2)$

b) $-3.25 \div 2.5$

c) $-5.5 \times (-5.5)$

d) $-4.37 \div (-0.95)$

7. Calculate. Express your answer to the nearest thousandth, if necessary.

a) $-2.4(-1.5)$ **b)** $8.6 \div 0.9$

c) $-5.3(4.2)$ **d)** $19.5 \div (-16.2)$

e) $1.12(0.68)$ **f)** $-0.55 \div 0.66$

For help with #8 to #11, refer to Example 3 on page 58.

8. Evaluate.

a) $-2.1 \times 3.2 + 4.3 \times (-1.5)$

b) $-3.5(4.8 - 5.6)$

c) $-1.1[2.3 - (-0.5)]$

Literacy Link

In $-1.1[2.3 - (-0.5)]$, square brackets are used for grouping because -0.5 is already in parentheses.

9. Determine each value.

a) $(4.51 - 5.32)(5.17 - 6.57)$

b) $2.4 + 1.8 \times 5.7 \div (-2.7)$

c) $-4.36 + 1.2[2.8 + (-3.5)]$

10. In Regina, Saskatchewan, the average mid-afternoon temperature in January is −12.6 °C. The average mid-afternoon temperature in July is 26.1 °C. How many degrees colder is Regina in January than in July?

11. One January day in Penticton, British Columbia, the temperature read −6.3 °C at 10:00 a.m. and 1.4 °C at 3:00 p.m.

a) What was the change in temperature?

b) What was the average rate of change in temperature?

Apply

12. A pelican dives vertically from a height of 3.8 m above the water. It then catches a fish 2.3 m underwater.

a) Write an expression using rational numbers to represent the length of the pelican's dive.

b) How long is the pelican's dive?

13. A submarine was cruising at a depth of 153 m. It then rose at 4.5 m/min for 15 min.

a) What was the submarine's depth at the end of this rise?

b) If the submarine continues to rise at the same rate, how much longer will it take to reach the surface?

14. Saida owned 125 shares of an oil company. One day, the value of each share dropped by $0.31. The next day, the value of each share rose by $0.18. What was the total change in the value of Saida's shares?

Literacy Link

A *share* is one unit of ownership in a corporation.

15. In dry air, the temperature decreases by about 0.65 °C for each 100-m increase in altitude.

a) The temperature in Red Deer, Alberta, is 10 °C on a dry day. What is the temperature outside an aircraft 2.8 km above the city?

b) The temperature outside an aircraft 1600 m above Red Deer is −8.5 °C. What is the temperature in the city?

16. Bella is more comfortable working with integers than with positive and negative decimal numbers. This is her way of understanding −4.3 + 2.5.

−4.3 is $\frac{-43}{10}$ or −43 tenths.

2.5 is $\frac{25}{10}$ or 25 tenths.

−43 tenths + 25 tenths is −18 tenths.

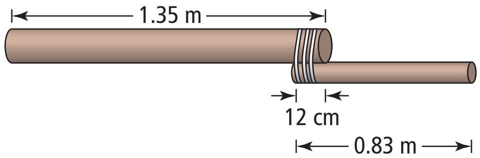
−18 tenths is $\frac{-18}{10}$ or −1.8.

So, −4.3 + 2.5 = −1.8.

a) Use Bella's method to determine 6.1 + (−3.9).

b) How could you modify Bella's method to determine 1.25 − 3.46?

17. Two wooden poles measured 1.35 m and 0.83 m in length. To make a new pole, they were attached by overlapping the ends and tying them together. The length of the overlap was 12 cm. What was the total length of the new pole in metres?



18. Determine the mean of each set of numbers. Express your answer to the nearest hundredth, if necessary.

a) 0, −4.5, −8.2, 0.4, −7.6, 3.5, −0.2

b) 6.3, −2.2, 14.9, −4.8, −5.3, 1.6

19. A company made a profit of $8.6 million in its first year. It lost $5.9 million in its second year and lost another $6.3 million in its third year.

a) What was the average profit or loss per year over the first three years?

b) The company broke even over the first four years. What was its profit or loss in the fourth year?

20. Research to find out the current price of gasoline in Calgary. It is 300 km from Calgary to Edmonton. How much less would it cost to drive this distance in a car with a fuel consumption of 5.9 L/100 km than in a car with a fuel consumption of 9.4 L/100 km? Give your answer in dollars, expressed to the nearest cent.

WWW Web Link

To find out prices of gas in Calgary, go to www.mathlinks9.ca and follow the links.

21. Andrew drove his car 234 km from Dawson to Mayo in Yukon Territory in 3 h. Brian drove his truck along the same route at an average speed of 5 km/h greater than Andrew's average speed. How much less time did Brian take, to the nearest minute?

22. An aircraft was flying at an altitude of 2950 m. It descended for 3 min at 2.5 m/s and then descended for 2.5 min at 2.8 m/s. What was the plane's altitude after the descent?

23. One week in October in Iqaluit, Nunavut, the daily high temperatures were −4.6 °C, −0.5 °C, 1.2 °C, 2.3 °C, −1.1 °C, 1.5 °C, and −3.0 °C. What was the mean daily high temperature that week?

Did You Know?

Iqaluit is the capital of Nunavut. This territory covers almost $\frac{1}{5}$ of the area of Canada but has less than $\frac{1}{1000}$ of the Canadian population. Over $\frac{4}{5}$ of the people who live in Nunavut are Inuit.

WWW Web Link

For more information about Nunavut, go to www.mathlinks9.ca and follow the links.

24. Copy and complete each statement.

a) ■ + 1.8 = −3.5

b) −13.3 − ■ = −8.9

c) ■ × (−4.5) = −9.45

d) −18.5 ÷ ■ = 7.4

25. Create a word problem that involves operations with rational numbers in decimal form. Make sure you can solve your problem. Then, have a classmate solve your problem.

Extend

26. Four points, A, B, C, and D, have the following coordinates:

A(0.75, 0.81)
B(0.75, −0.65)
C(−1.22, −0.65)
D(−1.22, 0.81)

What is the area of quadrilateral ABCD, to the nearest hundredth of a square unit?

27. The mean of six rational numbers is −4.3.

a) What is the sum of the rational numbers?

b) If five of the numbers each equal −4.5, what is the sixth number?

28. Evaluate each expression.

a) $3.6 + 2y$, $y = -0.5$

b) $(m - 1.8)(m + 1.8)$, $m = 1.7$

c) $\frac{4.5}{q} - \frac{q}{4.5}$, $q = -3.6$

29. Add one pair of parentheses or square brackets to the left side of each equation to make a true statement.

a) 3.5 × 4.1 − 3.5 − 2.8 = −0.7

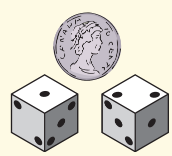
b) 2.5 + (−4.1) + (−2.3) × (−1.1) = 4.29

c) −5.5 − (−6.5) ÷ 2.4 + (−1.1) = −0.5

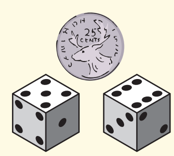
Math Link

Play this game with a partner or in a small group. You will need two dice and one coin.

- For each turn, roll the two dice and toss the coin. Then, repeat.
- Create numbers of the form ■.■ from the result of rolling the two dice.
- Tossing heads means the rational numbers are positive. Tossing tails means the rational numbers are negative.
- Record the two pairs of numbers.
- Choose one number from each pair so that the sum of the chosen numbers is as close as possible to zero. Record the sum of the chosen numbers.
- In each round, the player with the sum one closest to zero wins two points. If there is a tie, each tied player wins one point.
- The winner is the first player with ten points. If two or more players reach ten points in the same round, keep playing until one player is in the lead by at least two points.



gives the pair 1.2 and 2.1



gives the pair −5.6 and −6.5

The four possible sums are
1.2 + (−5.6)   1.2 + (−6.5)
2.1 + (−5.6)   2.1 + (−6.5)
Estimation shows that 2.1 + (−5.6) is closest to zero, so calculate this sum.
2.1 + (−5.6) = −3.5

2.3 Problem Solving With Rational Numbers in Fraction Form

Focus on...

After this lesson, you will be able to...
- **perform operations on rational numbers in fraction form**
- **solve problems involving rational numbers in fraction form**

Literacy Link

Klassen's winning time of 1:55.27 means 1 min, 55.27 s.

Web Link

To learn more about Cindy Klassen and other Canadian speed skaters, go to www.mathlinks9.ca and follow the links.

A news report gives the results of an Olympic speed skating event:

> Winnipeg's Cindy Klassen won the gold medal in the 1500-m speed skating event at the Winter Olympics in Turin, Italy. Her winning time was 1:55.27. Ottawa's Kristina Groves won the silver medal. She finished in a time of 1:56.74. The bronze medalist was Ireen Wust of the Netherlands. She finished only $\frac{16}{100}$ s behind Groves.

What are Klassen's time and Groves's time as a mixed number of seconds? How many seconds, in decimal form, did Wust finish behind Groves?

Explore Adding and Subtracting Rational Numbers

1. Determine the number of seconds by which Klassen beat Groves
   a) by using their times in fraction form to give an answer in fraction form
   b) by using their times in decimal form to give an answer in decimal form

2. Which method did you prefer in #1? Explain.

3. What was Wust's time for the event? Show your work.

4. By how many seconds did Klassen beat Wust? Use fractions to show two ways to determine your answer.

Reflect and Check

5. a) Use the following data to create a problem involving the addition or subtraction of rational numbers. Ask a partner to solve it.

> Canada's Perdita Felicien won in the 100-m hurdles at the world championships in Paris, France. Her time was 12.53 s. Brigitte Foster-Hylton of Jamaica placed second at 12.57 s. Foster-Hylton was $\frac{1}{10}$ s ahead of Miesha McKelvy of the United States.

b) Show how you could determine the answer to your problem by using a different method than your partner used.

c) Discuss with your partner the differences between your methods. Decide which method you prefer. Explain your choice.

Link the Ideas

Example 1: Add and Subtract Rational Numbers in Fraction Form

Estimate and calculate.

a) $\frac{2}{5} - \left(-\frac{1}{10}\right)$ **b)** $3\frac{2}{3} + \left(-1\frac{3}{4}\right)$

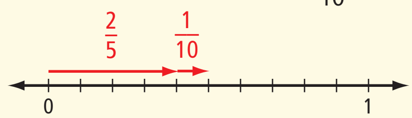
Solution

a) Estimate. $\frac{1}{2} - 0 = \frac{1}{2}$

Calculate.

$$\frac{2}{5} - \left(-\frac{1}{10}\right)$$
$$= \frac{2}{5} - \left(\frac{-1}{10}\right)$$
$$= \frac{4}{10} - \left(\frac{-1}{10}\right)$$
$$= \frac{4 - (-1)}{10}$$
$$= \frac{5}{10}$$
$$= \frac{1}{2}$$

Subtracting $-\frac{1}{10}$ is the same as adding the opposite of $-\frac{1}{10}$.



A common denominator of 5 and 10 is 10.

Is the calculated answer close to the estimate?

b) Estimate. $4 + (-2) = 2$

Calculate.

Method 1: Rewrite the Mixed Numbers as Improper Fractions

$$3\frac{2}{3} + \left(-1\frac{3}{4}\right) = \frac{11}{3} + \left(-\frac{7}{4}\right)$$

Add.

$$\frac{11}{3} + \left(-\frac{7}{4}\right)$$
$$= \frac{11}{3} + \left(\frac{-7}{4}\right)$$
$$= \frac{44}{12} + \left(\frac{-21}{12}\right)$$
$$= \frac{44 + (-21)}{12}$$
$$= \frac{23}{12}$$
$$= 1\frac{11}{12}$$

Method 2: Add the Integers and Add the Fractions

$$3\frac{2}{3} + \left(-1\frac{3}{4}\right)$$
$$= 3 + \frac{2}{3} + (-1) + \left(-\frac{3}{4}\right)$$
$$= 3 + (-1) + \frac{2}{3} + \left(-\frac{3}{4}\right)$$
$$= 3 + (-1) + \frac{8}{12} + \left(-\frac{9}{12}\right)$$
$$= 2 + \left(-\frac{1}{12}\right)$$
$$= 1\frac{12}{12} + \left(-\frac{1}{12}\right)$$
$$= 1\frac{11}{12}$$

Show You Know

Estimate and calculate.

a) $-\frac{3}{4} - \frac{1}{5}$

b) $-2\frac{1}{2} + 1\frac{9}{10}$

Example 2: Multiply and Divide Rational Numbers in Fraction Form

Determine.

a) $\frac{3}{4} \times \left(-\frac{2}{3}\right)$

b) $-1\frac{1}{2} \div \left(-2\frac{3}{4}\right)$

Solution

a) Multiply the numerators and multiply the denominators.

$$\begin{aligned}\frac{3}{4} \times \left(-\frac{2}{3}\right) &= \frac{3}{4} \times \left(\frac{-2}{3}\right)\\ &= \frac{3 \times (-2)}{4 \times 3}\\ &= \frac{-6}{12}\\ &= \frac{-1}{2} \text{ or } -\frac{1}{2}\end{aligned}$$

$1 \times \left(-\frac{1}{2}\right) = -\frac{1}{2}$ ME

You could remove the common factors of 3 and 2 from the numerator and denominator before multiplying.

$$\frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \left(\frac{\cancel{-2}^{-1}}{\cancel{3}_{1}}\right) = \frac{-1}{2} \text{ or } -\frac{1}{2}$$

b) *Method 1: Use a Common Denominator*

Write the fractions with a common denominator and divide the numerators.

$$\begin{aligned}-1\frac{1}{2} \div \left(-2\frac{3}{4}\right) &= -\frac{3}{2} \div \left(-\frac{11}{4}\right)\\ &= \frac{-6}{4} \div \left(\frac{-11}{4}\right)\\ &= \frac{-6}{-11}\\ &= \frac{6}{11}\end{aligned}$$

Recall that $\frac{-6}{4} \div \left(\frac{-11}{4}\right) = \frac{-6 \div (-11)}{4 \div 4} = \frac{-6 \div (-11)}{1} = \frac{-6}{-11}$

Method 2: Multiply by the Reciprocal

Another strategy is to multiply by the reciprocal.

$$\begin{aligned}\frac{-3}{2} \div \left(\frac{-11}{4}\right) &= \frac{-3}{2} \times \frac{4}{-11}\\ &= \frac{-12}{-22}\\ &= \frac{6}{11}\end{aligned}$$

You could remove the common factor of 2 from the numerator and denominator.

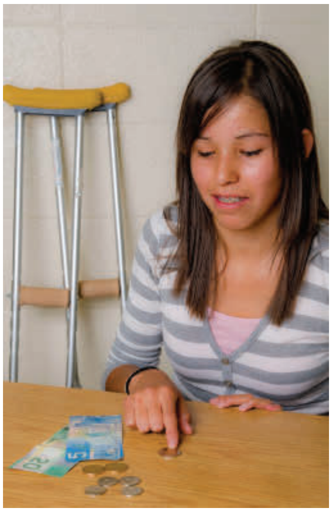
$$\frac{-3}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{-11} = \frac{6}{11}$$

Show You Know

Determine each value.

a) $-\frac{2}{5}\left(-\frac{1}{6}\right)$

b) $-2\frac{1}{8} \div 1\frac{1}{4}$



Example 3: Apply Operations With Rational Numbers in Fraction Form

At the start of a week, Maka had \$30 of her monthly allowance left. That week, she spent $\frac{1}{5}$ of the money on bus fares, another $\frac{1}{2}$ shopping, and $\frac{1}{4}$ on snacks. How much did she have left at the end of the week?

Solution

You can represent the \$30 Maka had at the beginning of the week by 30.

You can represent the fractions of the money spent by $-\frac{1}{5}$, $-\frac{1}{2}$, and $-\frac{1}{4}$.

Why would you represent the \$30 by a positive rational number?

Why would you represent the fractions of money spent by negative rational numbers?

Calculate each dollar amount spent.

For bus fares:

$$-\frac{1}{5} \times 30 = \frac{-1}{5} \times \frac{30}{1} = \frac{-30}{5} = -6$$

For shopping:

$$-\frac{1}{2} \times 30 = \frac{-1}{2} \times \frac{30}{1} = \frac{-30}{2} = -15$$

For snacks:

$$-\frac{1}{4} \times 30 = \frac{-1}{4} \times \frac{30}{1} = \frac{-30}{4} = -\frac{15}{2} \text{ or } -7.5$$

Determine the total dollar amount spent.
$-6 + (-15) + (-7.5) = -28.5$

Determine how much Maka had left.
$30 + (-28.5) = 1.5$

Maka had \$1.50 left at the end of the week.

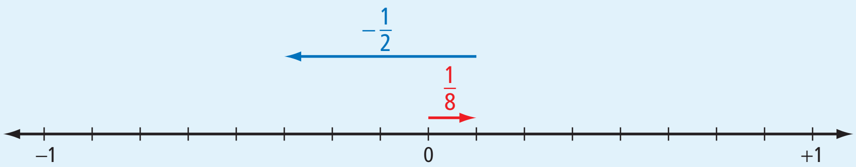
You could also calculate the total by adding the three fractions $-\frac{1}{5}$, $-\frac{1}{2}$, and $-\frac{1}{4}$ and multiplying their sum by 30. Why does this strategy work? Which strategy do you prefer?

Show You Know

Stefano had \$46 in a bank account that he was not using. Each month for three months, the bank withdrew $\frac{1}{4}$ of this amount as a service fee. How much was left in the account after the last withdrawal?

Key Ideas

- The addition of rational numbers can be modelled on a number line. The subtraction of rational numbers can be modelled by adding the opposite on a number line.



$$\frac{1}{8} + \left(-\frac{1}{2}\right) = -\frac{3}{8} \quad \text{or} \quad \frac{1}{8} - \frac{1}{2} = -\frac{3}{8}$$

- Rational numbers expressed as proper or improper fractions can be added and subtracted in the same way as positive fractions.
- Rational numbers expressed as mixed numbers can be added by
  - first writing them as improper fractions
  - adding the integers and adding the fractions
- Rational numbers expressed as mixed numbers can be subtracted by first writing them as improper fractions.
- Rational numbers expressed as proper or improper fractions can be multiplied and divided in the same way as positive fractions. The sign of the product or quotient can be predicted from the sign rules for multiplication and division.
- Rational numbers expressed as mixed numbers can be multiplied and divided by first writing them as improper fractions.

Check Your Understanding

Communicate the Ideas

1. Emma and Oleg both calculated $-\frac{1}{12} - \frac{2}{3}$ correctly.

a) Emma used 12 as a common denominator. Show how she calculated the difference in lowest terms.

b) Oleg used 36 as a common denominator. Show how he calculated the difference in lowest terms.

c) Which common denominator do you prefer using for the calculation? Explain.

2. Ming and Al both determined $-\frac{7}{15} \times \left(-\frac{5}{14}\right)$ and wrote the product in lowest terms. Ming multiplied before she removed common factors. Al removed common factors before he multiplied. Whose method do you prefer? Explain.

3. a) Calculate $\frac{-9}{12} \div \frac{3}{8}$ by multiplying by the reciprocal.

b) Calculate $\frac{-9}{12} \div \frac{3}{8}$ by writing the fractions with a common denominator and dividing the numerators.

c) Which method do you prefer for this calculation? Explain.

4. Joshua suggested his own method for multiplying and dividing rational numbers in fraction form. For $-\frac{4}{5} \times \frac{2}{3}$, he calculated $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$. Then, he reasoned that the product must be negative because $-\frac{4}{5}$ and $\frac{2}{3}$ have different signs. He gave the answer as $-\frac{8}{15}$. Describe an advantage and a disadvantage of Joshua's method.

Practise

For help with #5 and #6, refer to Example 1 on page 64. Write your answers in lowest terms.

5. Estimate and calculate.

a) $\frac{3}{10} + \frac{1}{5}$

b) $2\frac{1}{3} + \left(-1\frac{1}{4}\right)$

c) $-\frac{5}{12} - \frac{5}{12}$

d) $-2\frac{1}{2} - \left(-3\frac{1}{3}\right)$

e) $-\frac{5}{6} + \frac{1}{3}$

f) $\frac{3}{8} - \left(-\frac{1}{4}\right)$

6. Estimate and calculate.

a) $\frac{2}{3} - \frac{3}{4}$

b) $-\frac{2}{9} + \left(-\frac{1}{3}\right)$

c) $-\frac{1}{4} + \left(-\frac{3}{5}\right)$

d) $-\frac{3}{4} - \left(-\frac{5}{8}\right)$

e) $1\frac{1}{2} - 2\frac{1}{4}$

f) $1\frac{2}{5} + \left(-1\frac{3}{4}\right)$

For help with #7 and #8, refer to Example 2 on page 65. Write your answers in lowest terms.

7. Estimate and calculate.

a) $\frac{4}{5} \div \frac{5}{6}$

b) $3\frac{1}{3}\left(1\frac{3}{4}\right)$

c) $\frac{1}{8} \times \left(-\frac{2}{5}\right)$

d) $-\frac{9}{10} \div \left(-\frac{4}{5}\right)$

e) $-\frac{3}{8} \times 5\frac{1}{3}$

f) $\frac{1}{10} \div \left(-\frac{3}{8}\right)$

8. Estimate and calculate.

a) $-\frac{3}{4} \times \left(-\frac{1}{9}\right)$

b) $1\frac{1}{3} \div 1\frac{1}{4}$

c) $-\frac{3}{8} \div \frac{7}{10}$

d) $-2\frac{1}{8} \div 1\frac{1}{4}$

e) $\frac{7}{9}\left(-\frac{6}{11}\right)$

f) $-1\frac{1}{2} \div \left(-2\frac{1}{2}\right)$

For help with #9 and #10, refer to Example 3 on page 66.

9. Lori owed her mother \$39. Lori paid back $\frac{1}{3}$ of this debt and then paid back $\frac{1}{4}$ of the remaining debt.

How much does Lori still owe her mother?

10. A carpenter has 64 m of baseboard. He installs $\frac{1}{2}$ of the baseboard in one room. He installs another $\frac{3}{5}$ of the original amount of baseboard in another room. How much baseboard does he have left?

Apply

11. In everyday speech, *in a jiffy* means in a very short time. In science, a specific value sometimes assigned to a jiffy is $\frac{1}{100}$ s. Naima can type at 50 words/min. On average, how many jiffies does she take to type each word?

12. In the table, a positive number shows how many hours the time in a location is ahead of the time in London, England. A negative number shows how many hours the time is behind the time in London.

Location	Time Zone
Alice Springs, Australia	$+9\frac{1}{2}$
Brandon, Manitoba	-6
Chatham Islands, New Zealand	$+12\frac{3}{4}$
Istanbul, Turkey	$+2$
Kathmandu, Nepal	$+5\frac{3}{4}$
London, England	0
Mumbai, India	$+5\frac{1}{2}$
St. John's, Newfoundland and Labrador	$-3\frac{1}{2}$
Tokyo, Japan	$+9$
Victoria, British Columbia	-8

a) How many hours is the time in St. John's ahead of the time in Brandon?

b) How many hours is the time in Victoria behind the time in Mumbai?

c) Determine and interpret the time difference between Tokyo and Kathmandu.

d) Determine and interpret the time difference between Chatham Islands and St. John's.

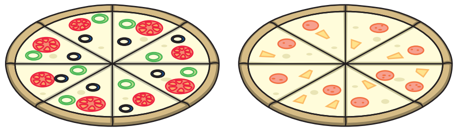
e) In which location is the time exactly halfway between the times in Istanbul and Alice Springs?

13. The diameter of Pluto is $\frac{6}{17}$ the diameter of Mars. Mars is $\frac{17}{300}$ the diameter of Saturn.

a) What fraction of the diameter of Saturn is the diameter of Pluto?

b) The diameter of Saturn is 120 000 km. What is the diameter of Pluto?

14. Li and Ray shared a vegetarian pizza and a Hawaiian pizza of the same size. The vegetarian pizza was cut into eight equal slices. The Hawaiian pizza was cut into six equal slices. Li ate two slices of the vegetarian pizza and one slice of the Hawaiian pizza. Ray ate two slices of the Hawaiian pizza and one slice of the vegetarian pizza.



a) Who ate more pizza?

b) How much more did that person eat?

c) How much pizza was left over?

15. Predict the next three numbers in each pattern.

a) $-1\frac{1}{2}, -\frac{7}{8}, -\frac{1}{4}, \frac{3}{8}, 1, \ldots$

b) $1\frac{1}{3}, -\frac{2}{3}, \frac{1}{3}, -\frac{1}{6}, \frac{1}{12}, \ldots$

16. Boris has $2\frac{1}{2}$ times as much cash as Anna. Charlie has $\frac{3}{4}$ as much cash as Anna. Anna has \$25.60 in cash.

a) How much cash do the three people have altogether?

b) How much more cash does Boris have than Charlie?

17. To calculate $-\frac{3}{4} + \left(-\frac{2}{3}\right)$, Amy decided to convert the fractions to decimals and add the decimals on a scientific calculator.

a) Explain why she had difficulty in determining the exact answer by this method.

b) How should she calculate to get an exact answer?

18. One ninth of the height of an iceberg was above the surface of the water. The iceberg extended to a depth of 75.8 m below the surface. What was the height of the iceberg above the surface? Express your answer to the nearest tenth of a metre.

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19. Copy and complete each statement.

a) $\frac{1}{2} + \blacksquare = -\frac{3}{4}$

b) $\blacksquare - 1\frac{4}{5} = -\frac{7}{10}$

c) $-2\frac{1}{6} \times \blacksquare = -1\frac{1}{3}$

d) $\blacksquare \div \left(-\frac{3}{5}\right) = 2\frac{1}{2}$

20. In a magic square, the sum of each row, column, and diagonal is the same. Copy and complete this magic square.

$-\frac{1}{2}$		
	$-\frac{5}{6}$	
	$\frac{1}{2}$	$-1\frac{1}{6}$

21. Calculate.

a) $\frac{1}{3}\left(\frac{2}{5} - \frac{1}{2}\right) + \frac{3}{10}$

b) $\frac{3}{4} \div \frac{5}{8} - \frac{3}{8} \div \frac{1}{2}$

c) $1\frac{1}{2} + 1\frac{1}{2}\left(-2\frac{5}{6} + \frac{1}{3}\right)$

22. Taj has three scoops for measuring flour. The largest scoop holds $2\frac{1}{2}$ times as much as the smallest one. The middle scoop holds $1\frac{3}{4}$ times as much as the smallest one. Describe two different ways in which Taj could measure each of the following quantities. He can use full scoops only.

a) $3\frac{1}{4}$ times as much as the smallest scoop holds

b) $\frac{1}{2}$ as much as the smallest scoop holds

23. a) Write a subtraction statement involving two negative fractions or negative mixed numbers so that the difference is $-\frac{4}{3}$.

b) Share your statement with a classmate.

Extend

24. Can the sum of two rational numbers be less than both of the rational numbers? Explain using examples in fraction form.

25. The following expression has a value of 1.

$$\left[-\frac{1}{2} + \left(-\frac{1}{2}\right)\right] \div \left[-\frac{1}{2} + \left(-\frac{1}{2}\right)\right]$$

Use $-\frac{1}{2}$ four times to write expressions with each of the following values.

a) -1

b) 0

c) $\frac{1}{4}$

d) 4

e) $-\frac{3}{4}$

f) $-1\frac{1}{2}$

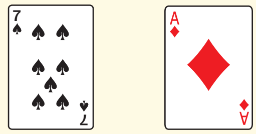
26. Multiplying a fraction by $-\frac{1}{2}$, then adding $\frac{3}{4}$, and then dividing by $-\frac{1}{4}$ gave an answer of $-3\frac{3}{4}$. What was the original fraction?

27. For what values of x does $x - \frac{1}{x} = 1\frac{1}{2}$?

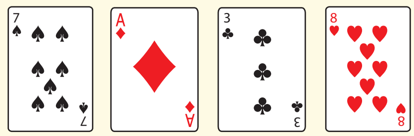
Math Link

Play this game with a partner or in a small group. You will need a deck of playing cards.

- Remove the jokers, face cards, and 10s from the deck.
- Red cards represent positive integers. Black cards represent negative integers. Aces represent 1 or −1.
- In each round, the dealer shuffles the cards and deals four cards to each player.
- Use your four cards to make two fractions with a product that is as far from zero as possible.
- In each round, the player with the product that is furthest from zero wins two points. If there is a tie, each tied player wins a point.
- The winner is the first player with ten points. If two or more players reach ten points in the same round, keep playing until one player is in the lead by at least two points.



represents −7 represents 1



represent −7, 1, −3, and 8

An expression for the product furthest from zero is

$\frac{-7}{1} \times \frac{8}{-3}$ or $\frac{8}{-3} \times \frac{-7}{1}$ or $\frac{8}{1} \times \frac{-7}{-3}$ or $\frac{-7}{-3} \times \frac{8}{1}$

History Link

Fractions in Ancient Egypt

A fraction with a numerator of 1, such as $\frac{1}{4}$, is called a *unit fraction*. In ancient Egypt, fractions that were not unit fractions were expressed as sums of unit fractions. For example, $\frac{5}{6}$ was expressed as $\frac{1}{2} + \frac{1}{3}$.

1. Express each of the following as the sum of two unit fractions.

a) $\frac{3}{10}$ **b)** $\frac{9}{14}$ **c)** $\frac{9}{20}$ **d)** $\frac{11}{18}$

2. Describe any strategies that helped you to complete #1.

Did You Know?

The Eye of Horus as shown below was used by ancient Egyptians to represent fractions. Each part of the Eye of Horus was a unit fraction. Egyptians believed that the parts had a combined value of 1.

Repetition was not allowed in the Egyptian system. Therefore, $\frac{2}{5}$ was not expressed as $\frac{1}{5} + \frac{1}{5}$ but could be expressed as $\frac{1}{15} + \frac{1}{3}$.

3. Express each of the following as the sum of two unit fractions without using any fraction more than once.

a) $\frac{2}{7}$ **b)** $\frac{2}{9}$ **c)** $\frac{2}{11}$

4. Express each of the following as the sum of three unit fractions without using any fraction more than once.

a) $\frac{7}{8}$ **b)** $\frac{11}{24}$ **c)** $\frac{3}{4}$

2.4 Determining Square Roots of Rational Numbers

Focus on...

After this lesson, you will be able to...
- determine the square root of a perfect square rational number
- determine an approximate square root of a non-perfect square rational number

The Great Pyramid of Giza is the largest of the ancient pyramids in Egypt. The pyramid has a square base with a side length between 230 m and 231 m. Estimate how the dimensions of the base compare with the dimensions of a football field.

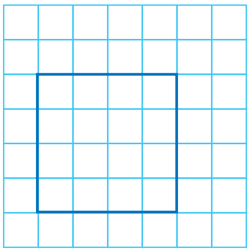
Materials
- grid paper

Literacy Link

When the *square root* of a given number is multiplied by itself, the product is the given number. For example, the square root of 9 is 3, because $3 \times 3 = 9$. A square root is represented by the symbol $\sqrt{\ }$, for example, $\sqrt{9} = 3$.

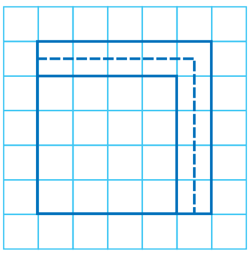
Explore Square Roots of Rational Numbers

1. a) Explain how the diagram represents $\sqrt{16}$.



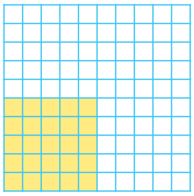
b) Draw a diagram that represents $\sqrt{25}$.

c) Explain how you could use the following diagram to identify a rational number with a square root that is between 4 and 5.



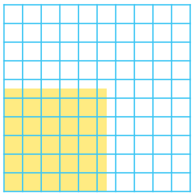
d) Describe another strategy you could use to complete part c).

2. a) Explain how the shading on the hundred grid represents $\sqrt{0.25}$.



b) Draw a diagram that represents $\sqrt{0.36}$.

c) Explain how you could use the following diagram to identify a rational number with a square root that is between 0.5 and 0.6.

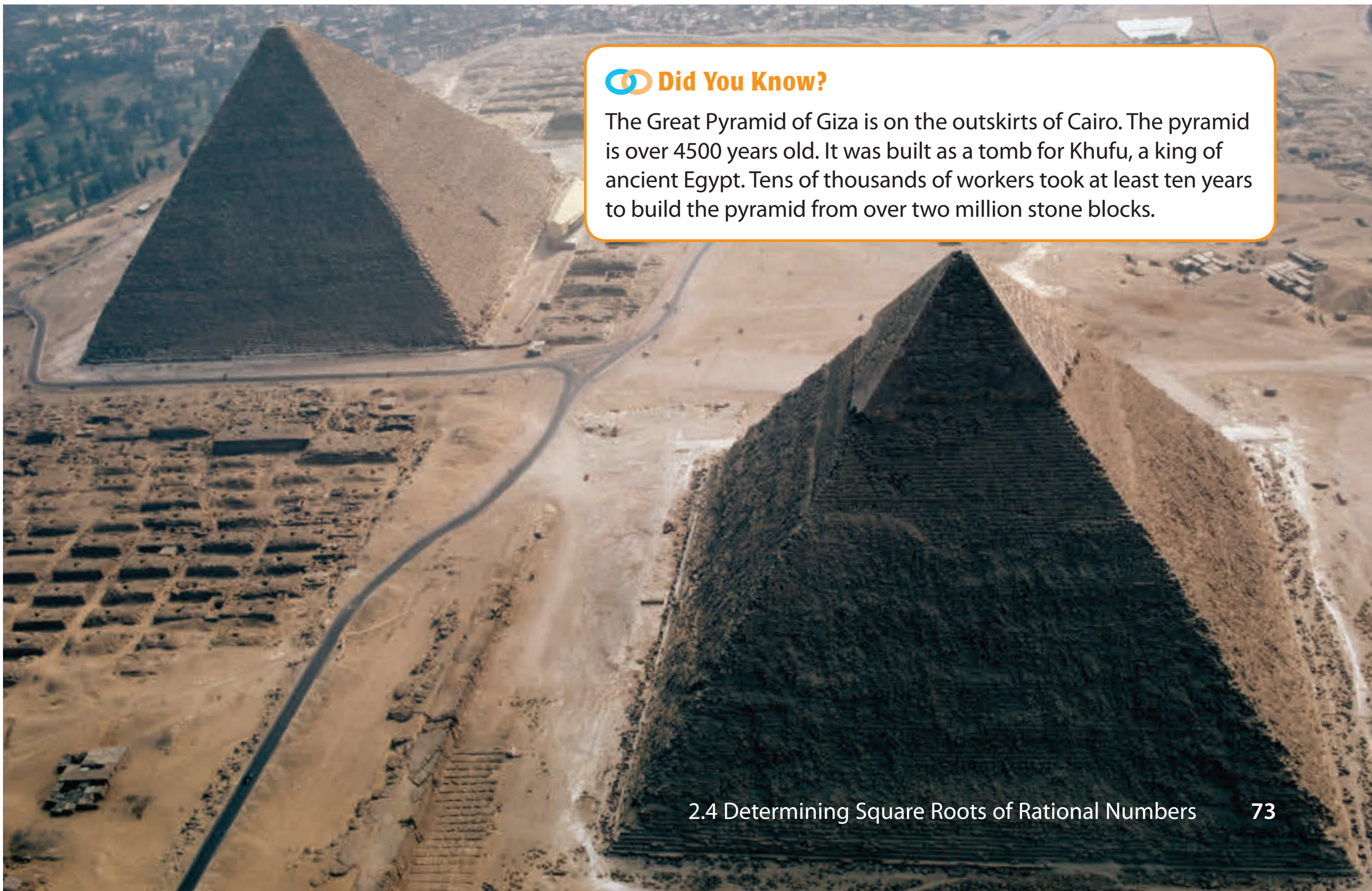


d) Describe another strategy you could use to complete part c).

Reflect and Check

3. Compare your strategies from #1d) and #2d) with a classmate's strategies. How are they similar and different?

4. Use the dimensions provided in the opening paragraph of this section to estimate the base area of the Great Pyramid of Giza. Explain your method.



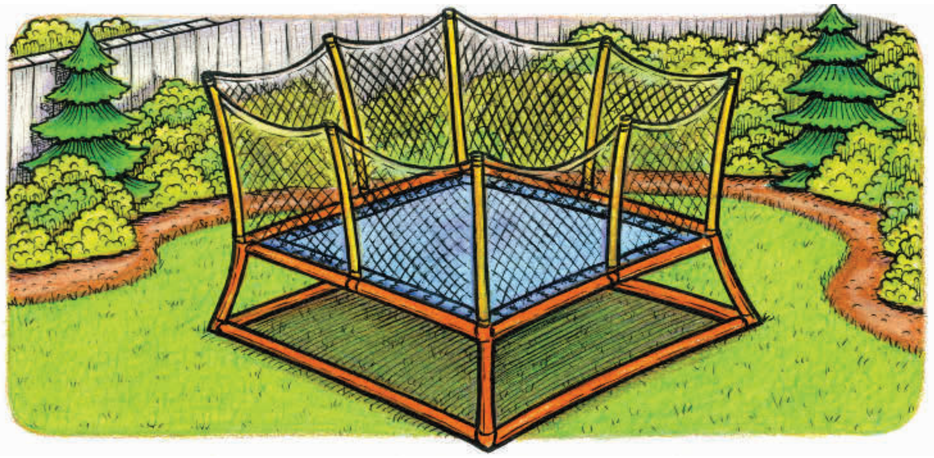
Did You Know?

The Great Pyramid of Giza is on the outskirts of Cairo. The pyramid is over 4500 years old. It was built as a tomb for Khufu, a king of ancient Egypt. Tens of thousands of workers took at least ten years to build the pyramid from over two million stone blocks.

Link the Ideas

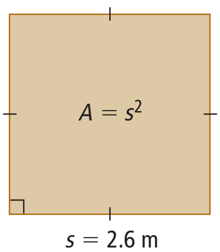
Example 1: Determine a Rational Number From Its Square Root

A square trampoline has a side length of 2.6 m. Estimate and calculate the area of the trampoline.



Solution

Strategies
Draw a Diagram



Estimate.
$2^2 = 4$ $3^2 = 9$
So, 2.6^2 is between 4 and 9.
2.6 is closer to 3 than to 2, so $2.6^2 \approx 7$.
An estimate for the area of the trampoline is 7 m^2.

Calculate.
$2.6^2 = 6.76$ C 2.6 x^2 6.76
The area of a trampoline with a side length of 2.6 m is 6.76 m^2.

Tech Link

Check the key sequence for determining the square of a number on your calculator. If there is no x^2 or equivalent key, just multiply the number by itself.

Show You Know

Estimate and calculate the area of a square photo with a side length of 7.1 cm.

Example 2: Determine Whether a Rational Number Is a Perfect Square

Determine whether each of the following numbers is a perfect square.

a) $\frac{25}{49}$ **b)** 0.4

Solution

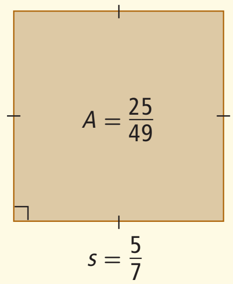
a) In $\frac{25}{49}$, both the numerator and denominator are perfect squares.

How do you know that 25 and 49 are perfect squares?

$\frac{25}{49}$ can be expressed as the product of two equal rational factors, $\frac{5}{7} \times \frac{5}{7}$.

So, $\frac{25}{49}$ is a perfect square.

How does this diagram represent the situation?



b) 0.4 can be expressed in fraction form as $\frac{4}{10}$.

The numerator, 4, is a perfect square.
The denominator, 10, is not a perfect square.

How do you know that 10 is not a perfect square?

$\frac{4}{10}$ cannot be expressed as the product of two equal rational factors.

So, 0.4 is not a perfect square.

Literacy Link

A *perfect square* can be expressed as the product of two equal rational factors. The decimal 0.25 is a perfect square because it can be expressed as 0.5×0.5. The fraction $\frac{9}{16}$ is a perfect square because it can be expressed as $\frac{3}{4} \times \frac{3}{4}$.

Show You Know

Is each of the following numbers a perfect square? Explain.

a) $\frac{121}{64}$ **b)** 1.2 **c)** 0.09

Literacy Link

Square Roots of Perfect Squares

0.25 can be expressed as 0.5×0.5.

Therefore, $\sqrt{0.25} = 0.5$.

$\frac{9}{16}$ can be expressed as $\frac{3}{4} \times \frac{3}{4}$.

Therefore, $\sqrt{\frac{9}{16}} = \frac{3}{4}$.

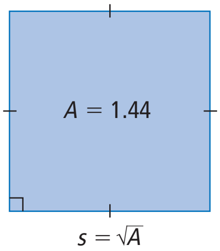
Strategies

Draw a Diagram

Example 3: Determine the Square Root of a Perfect Square

Evaluate $\sqrt{1.44}$.

Solution



Determine the positive number that, when multiplied by itself, results in a product of 1.44.

Method 1: Use Inspection

$1.2 \times 1.2 = 1.44$

So, $\sqrt{1.44} = 1.2$.

Since $12 \times 12 = 144$, then $1.2 \times 1.2 = 1.44$.

Method 2: Use Guess and Check

$1.1 \times 1.1 = 1.21$	Too low
$1.3 \times 1.3 = 1.69$	Too high
$1.2 \times 1.2 = 1.44$	Correct!

So, $\sqrt{1.44} = 1.2$.

Method 3: Use Fraction Form

$$1.44 = \frac{144}{100}$$
$$= \frac{12}{10} \times \frac{12}{10}$$
$$= 1.2 \times 1.2$$

So, $\sqrt{1.44} = 1.2$.

Check: C **1.44** $\sqrt{x}$ 1.2

Tech Link

Check the key sequence for determining square roots on your calculator. Make sure that you can obtain the correct answer for Example 3.

Show You Know

Evaluate.

a) $\sqrt{2.25}$

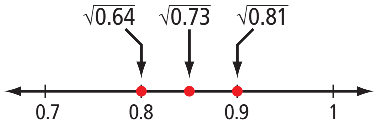
b) $\sqrt{0.16}$

Example 4: Determine a Square Root of a Non-Perfect Square

a) Estimate $\sqrt{0.73}$.
b) Calculate $\sqrt{0.73}$, to the nearest thousandth.

Solution

a) Estimate.
You can use the square root of a perfect square on each side of $\sqrt{0.73}$.
$\sqrt{0.73}$ is about halfway between $\sqrt{0.64}$ and $\sqrt{0.81}$.



One reasonable estimate for $\sqrt{0.73}$ might be about halfway between 0.8 and 0.9, which is about 0.85.

$\sqrt{0.73} \approx 0.85$

b) Calculate.
C 0.73 √x 0.854400375
So, $\sqrt{0.73} \approx 0.854$, to the nearest thousandth.

A calculator usually gives a closer approximation than an estimate does. Why is the value that the calculator shows only an approximation?

Check:
Use the inverse operation, which is squaring.
$0.854^2 = 0.729\ 316$
0.854^2 is close to 0.73.

Show You Know

a) Estimate $\sqrt{0.34}$.
b) Calculate $\sqrt{0.34}$, to the nearest thousandth.

non-perfect square

- a rational number that cannot be expressed as the product of two equal rational factors
- for example, you cannot multiply any rational number by itself and get an answer of 3, 5, 1.5, or $\frac{7}{8}$

Strategies
Draw a Diagram

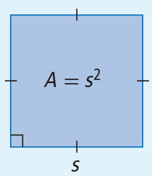
Did You Know?

The square root of a non-perfect square has certain properties:

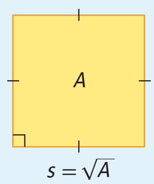
- It is a non-repeating, non-terminating decimal. For example, $\sqrt{5} = 2.236\ 067\ 978\ldots$.
- Its decimal value is approximate, not exact.

Key Ideas

- If the side length of a square models a number, the area of the square models the square of the number.



- If the area of a square models a number, the side length of the square models the square root of the number.



- A perfect square can be expressed as the product of two equal rational factors. $3.61 = 1.9 \times 1.9$ $\quad \frac{1}{4} = \frac{1}{2} \times \frac{1}{2}$
- The square root of a perfect square can be determined exactly. $\sqrt{2.56} = 1.6$ $\quad \sqrt{\frac{4}{9}} = \frac{2}{3}$
- The square root of a non-perfect square determined using a calculator is an approximation. $\sqrt{1.65} \approx 1.284\ 523\ 258$

Check Your Understanding

Communicate the Ideas

1. Max said that the square root of 6.4 is 3.2. Lynda claimed that the square root of 6.4 is 0.8. Jamila estimated that the square root of 6.4 should be about 2.5.

a) Who is correct?

b) What mistakes did the other two students make?

2. Without calculating any square roots, identify the square roots that have values between 4.5 and 5.5. Explain your reasoning.

$\sqrt{21.3}$ $\sqrt{20.1}$ $\sqrt{31.7}$ $\sqrt{27.9}$ $\sqrt{30.5}$ $\sqrt{25.4}$ $\sqrt{30.2}$

3. Since $\sqrt{9}$ is less than 9, and $\sqrt{1.44}$ is less than 1.44, André predicted that $\sqrt{0.0625}$ would be less than 0.0625. Do you agree with his prediction? Explain.

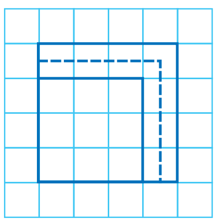
4. a) Determine $\sqrt{2}$ using a scientific calculator. Record all of the digits that the calculator displays.

b) Enter the decimal value of $\sqrt{2}$ from part a) into the calculator and square the value. Record the result.

c) In part a), did the calculator display the exact value of $\sqrt{2}$? Explain how you know.

Practise

5. Use the diagram to identify a rational number with a square root between 3 and 4.



6. Identify a rational number with a square root between 0.7 and 0.8.

For help with #7 and #8, refer to Example 1 on page 74.

7. Estimate and calculate the number that has the given square root.

a) 3.1 **b)** 12.5

c) 0.62 **d)** 0.29

8. Estimate and calculate the area of each square, given its side length.

a) 4.3 cm **b)** 0.035 km

For help with #9 and #10, refer to Example 2 on page 75.

9. Is each of the following rational numbers a perfect square? Explain.

a) $\frac{1}{16}$ **b)** $\frac{5}{9}$

c) 0.36 **d)** 0.9

10. Determine whether each rational number is a perfect square.

a) $\frac{7}{12}$ **b)** $\frac{100}{49}$

c) 0.1 **d)** 0.01

For help with #11 and #12, refer to Example 3 on page 76.

11. Evaluate.

a) $\sqrt{324}$ **b)** $\sqrt{2.89}$

c) $\sqrt{0.0225}$ **d)** $\sqrt{2025}$

12. Calculate the side length of each square from its area.

a) 169 m^2 **b)** 0.16 mm^2

For help with #13 and #14, refer to Example 4 on page 77.

13. Estimate each square root. Then, calculate it to the specified number of decimal places.

a) $\sqrt{39}$, to the nearest tenth

b) $\sqrt{4.5}$, to the nearest hundredth

c) $\sqrt{0.87}$, to the nearest thousandth

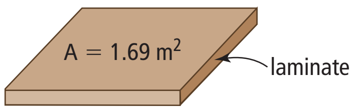
d) $\sqrt{0.022}$, to the nearest thousandth

14. Given the area of each square, determine its side length. Express your answer to the nearest hundredth of a unit.

a) 0.85 m^2 **b)** 60 cm^2

Apply

15. Kai needs to replace the strip of laminate that is glued to the vertical faces on a square tabletop. The tabletop has an area of 1.69 m^2. What length of laminate does she need?



16. a) The label on a 1-L can of paint states that the paint will cover an area of 10 m^2. What is the side length of the largest square area that the paint will cover? Express your answer to the nearest hundredth of a metre.

b) What is the side length of the largest square area that a 3.79-L can of the same paint will cover? Express your answer to the nearest hundredth of a metre.

c) Nadia is applying two coats of the paint to an area that is 4.6 m by 4.6 m. How much paint will she use if she applies the same amount of paint for each coat? Express your answer to the nearest tenth of a litre.

17. Some parks contain fenced gardens. Suppose that it costs \$80 to build each metre of fence, including materials and labour.

a) How much does it cost to enclose a square with an area of 120 m^2? Express your answer to the nearest dollar.

b) Predict whether the total cost of enclosing two squares with an area of 60 m^2 each is the same as your answer to part a).

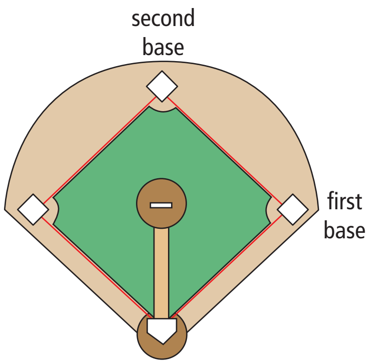
c) Test your prediction from part b) and describe your findings.

18. A frame measures 30 cm by 20 cm. Can you mount a square picture with an area of 500 cm^2 in the frame? Explain.

19. A square picture with an area of 100 cm^2 is mounted on a square piece of matting. The matting has 2.5 times the area of the picture. If the picture is centred on the matting, what width of matting is visible around the outside of the picture? Give your answer to the nearest tenth of a centimetre.

20. Leon's rectangular living room is 8.2 m by 4.5 m. A square rug covers $\frac{2}{5}$ of the area of the floor. What is the side length of the rug, to the nearest tenth of a metre?

21. A baseball diamond is a square area of about 750 m^2. What is the distance from first to second base. Give your answer to the nearest tenth of a metre.



22. The hypotenuse of an isosceles right triangle has a length of 20 cm. What is the length of each leg of the triangle? Provide your answer to the nearest tenth of a centimetre.

23. A rectangular floor that measures 3 m by 2 m is covered by 384 square tiles. Determine the side length of each tile, in centimetres. State any assumptions you make.

24. The distance, d, in kilometres, that a person can see across the ocean to the horizon is given by the formula $d = \sqrt{12.74 \times h}$. In the formula h is the height, in metres, of the person's eyes above the water. Calculate the distance that each of the following people can see across the ocean to the horizon. Express each answer to the nearest tenth of a kilometre.

a) Adèle is sitting on a lifeguard station at the edge of the ocean. Her eyes are 4.1 m above the water.

b) Brian is standing at the water's edge. His eyes are 165 cm above the water.

c) Yvonne is the pilot of an aircraft flying 5 km above the coastline.

Literacy Link

Perform operations under a square root symbol before taking the square root. For example, $\sqrt{9 \times 4} = \sqrt{36}$ or 6.

25. What is the length of the longest line segment you can draw on a sheet of paper that is 27.9 cm by 21.6 cm? Express your answer to the nearest tenth of a centimetre.

26. A bag of fertilizer will cover an area of 200 m^2. Determine the dimensions of a square that $\frac{3}{4}$ of a bag of fertilizer will cover. Express your answer to the nearest tenth of a metre.

27. The surface area of a cube is 100 cm^2. Determine the edge length of the cube, to the nearest tenth of a centimetre.

28. The period, t, in seconds, of a pendulum is the time it takes for a complete swing back and forth. The period can be calculated from the length, l, in metres, of the pendulum using the formula $t = \sqrt{4l}$. Determine the period of a pendulum with each of the following lengths. Express each answer to the nearest hundredth of a second.

a) 1.6 m **b)** 2.5 m **c)** 50 cm

29. The speed of sound in air, s, in metres per second, is related to the temperature, t, in degrees Celsius, by the formula $s = \sqrt{401(273 + t)}$. How much greater is the speed of sound on a day when the temperature is 30 °C than on a day when the temperature is −20 °C? Express your answer to the nearest metre per second.

30. A square field has an area of 1000 m^2. Laura wants to walk from one corner of the field to the opposite corner. If she walks at 1.5 m/s, how much time can she save by walking diagonally instead of walking along two adjacent sides? Express your answer to the nearest tenth of a second.

31. The area of a triangle can be determined using Heron's formula, which requires the side lengths of the triangle. Heron's formula is $A = \sqrt{s(s - a)(s - b)(s - c)}$. In the formula A is the area; a, b, and c are the side lengths; and s is half the perimeter or $\frac{a + b + c}{2}$. Determine the area of each triangle with the following side lengths. Express each area to the nearest tenth of a square centimetre.

a) 15 cm, 12 cm, 10 cm

b) 9.3 cm, 11.4 cm, 7.5 cm

History Link

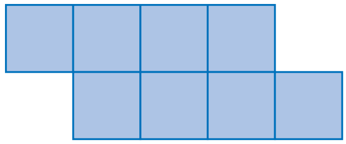
Heron's formula was determined by a Greek mathematician and inventor named Heron (or Hero) of Alexandria. Historians believe that he lived in the first century of the Common Era, but little is known of his life. Heron wrote about geometry and about his inventions, which included machines driven by water, steam, and compressed air.

WWW Web Link

For more information about Heron of Alexandria, go to www.mathlinks9.ca and follow the links.

Extend

32. This shape is made from eight congruent squares. The total area of the shape is 52 cm². What is its perimeter, to the nearest centimetre?



33. A square has an area of 32 cm². What is the area of the largest circle that will fit inside it? Express your answer to the nearest tenth of a square centimetre.

34. Use the formula $r = \sqrt{\frac{A}{\pi}}$ to determine the radius of a circular garden with an area of 40 m². Express your answer to the nearest tenth of a metre.

35. The width of a rectangle is $\frac{1}{3}$ its length. The area of the rectangle is 9.72 cm². What are the dimensions of the rectangle?

36. Determine $\sqrt{\sqrt{\sqrt{65\ 536}}}$.

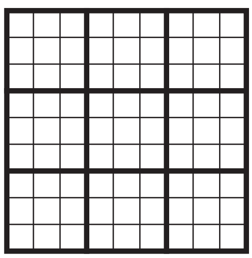
Math Link

Sudoku is a Japanese logic puzzle completed on a 9-by-9 square grid. The grid includes nine 3-by-3 sections.

Answer each of the following questions about the sudoku grid in two different ways. Compare your solutions with your classmates' solutions.

a) If the smallest squares on the grid have a side length of 1.1 cm, what is the area of the whole grid?

b) If the whole grid has an area of 182.25 cm², what are the dimensions of each 3 by 3 section?



WWW Web Link

To learn more about sudoku puzzles, go to www.mathlinks9.ca and follow the links.

Chapter 2 Review

For #1 to #4, use the clues to unscramble the letters.

1. S T I S P O P O E

two numbers represented by points that are the same distance in opposite directions from zero on a number line

2. T A L I N A R O B R U N M E

the quotient of two integers, where the divisor is not zero (2 words)

3. C R E F P E T Q U E S A R

the product of two equal rational factors (2 words)

4. F R E N C E N T O P A Q U E R S

a rational number that cannot be expressed as the product of two equal rational factors (2 words, 1 hyphenated)

2.1 Comparing and Ordering Rational Numbers, pages 46–54

5. Which of the following rational numbers cannot be expressed as an integer?

$\frac{24}{3}$ $\frac{3}{24}$ $\frac{-8}{2}$ $\frac{-10}{-6}$ $-\frac{6}{4}$

$-\left(\frac{-21}{-7}\right)$ $\frac{82}{-12}$ $-\left(\frac{-225}{15}\right)$

6. Replace each ■ with >, <, or = to make each statement true.

a) $\frac{-9}{6}$ ■ $\frac{3}{-2}$

b) -0.86 ■ -0.84

c) $-\frac{3}{5}$ ■ $-0.\overline{6}$

d) $-1\frac{3}{10}$ ■ $-\left(\frac{-13}{-10}\right)$

e) $-\frac{8}{12}$ ■ $-\frac{11}{15}$

f) $-2\frac{5}{6}$ ■ $-2\frac{7}{8}$

7. Axel, Bree, and Caitlin were comparing $-1\frac{1}{2}$ and $-1\frac{1}{4}$.

a) Axel first wrote the two mixed numbers as improper fractions. Describe the rest of his method.

b) Bree first wrote each mixed number as a decimal. Describe the rest of her method.

c) Caitlin first ignored the integers and wrote $-\frac{1}{2}$ and $-\frac{1}{4}$ with a common denominator. Describe the rest of her method.

d) Which method do you prefer? Explain.

8. Write two fractions in lowest terms between 0 and −1 with 5 as the numerator.

2.2 Problem Solving With Rational Numbers in Decimal Form, pages 55–62

9. Calculate.

a) $-5.68 + 4.73$

b) $-0.85 - (-2.34)$

c) $1.8(-4.5)$

d) $-3.77 \div (-2.9)$

10. Evaluate. Express your answer to the nearest tenth, if necessary.

a) $5.3 \div (-8.4)$

b) $-0.25 \div (-0.031)$

c) $-5.3 + 2.4[7.8 + (-8.3)]$

d) $4.2 - 5.6 \div (-2.8) - 0.9$

11. One evening in Dauphin, Manitoba, the temperature decreased from 2.4 °C to −3.2 °C in 3.5 h. What was the average rate of change in the temperature?

12. Over a four-year period, a company lost an average of $1.2 million per year. The company's total losses by the end of five years were $3.5 million. What was the company's profit or loss in the fifth year?

2.3 Problem Solving With Rational Numbers in Fraction Form, pages 63–71

13. Add or subtract.

a) $\frac{2}{3} - \frac{4}{5}$ **b)** $-\frac{3}{8} + \left(-\frac{3}{4}\right)$

c) $-3\frac{3}{5} + 1\frac{7}{10}$ **d)** $2\frac{1}{3} - \left(-2\frac{1}{4}\right)$

14. Multiply or divide.

a) $-\frac{1}{2}\left(-\frac{8}{9}\right)$ **b)** $-\frac{5}{6} \div \frac{7}{8}$

c) $2\frac{3}{4} \times \left(-4\frac{2}{3}\right)$ **d)** $-4\frac{7}{8} \div \left(-2\frac{3}{4}\right)$

15. Without doing any calculations, state how the values of the following two quotients compare. Explain your reasoning.

$96\frac{7}{8} \div 7\frac{3}{4}$ $\qquad -96\frac{7}{8} \div \left(-7\frac{3}{4}\right)$

16. How many hours are there in $2\frac{1}{2}$ weeks?

17. The area of Manitoba is about $1\frac{1}{5}$ times the total area of the four Atlantic provinces. The area of Yukon Territory is about $\frac{3}{4}$ the area of Manitoba. Express the area of Yukon Territory as a fraction of the total area of the Atlantic provinces.

2.4 Determining Square Roots of Rational Numbers, pages 72–81

18. Determine whether each rational number is a perfect square. Explain your reasoning.

a) $\frac{64}{121}$ **b)** $\frac{7}{4}$ **c)** 0.49 **d)** 1.6

19. Estimate $\sqrt{220}$ to one decimal place. Describe your method.

20. Determine the number with a square root of 0.15.

21. Determine.

a) $\sqrt{12.96}$

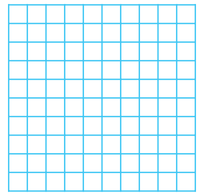
b) $\sqrt{0.05}$, to the nearest thousandth

22. In what situation is each of the following statements true? Provide an example to support each answer.

a) The square root of a number is less than the number.

b) The square root of a number is greater than the number.

23. A hundred grid has an area of 225 cm^2.


$A = 225\ cm^2$

a) What is the side length of each small square on the grid? Solve this problem in two ways.

b) What is the length of the diagonal of the whole grid? Express your answer to the nearest tenth of a centimetre.

24. Suppose a 1-L can of paint covers 11 m^2.

a) How many cans of paint would you need to paint a ceiling that is 5.2 m by 5.2 m? Show your work.

b) Determine the maximum dimensions of a square ceiling you could paint with 4 L of paint. Express your answer to the nearest tenth of a metre.

25. Close to the surface of the moon, the time a dropped object takes to reach the surface can be determined using the formula $t = \sqrt{\frac{h}{0.81}}$. The time, *t*, is in seconds, and the height, *h*, is in metres. If an object is dropped from a height of 200 m, how long does it take to reach the surface of the moon? Express your answer to the nearest tenth of a second.

Chapter 2 Practice Test

For #1 to #7, select the best answer.

1. Which fraction does not equal $\frac{4}{-6}$?

A $-\left(\frac{-10}{15}\right)$ **B** $-\frac{8}{12}$

C $\frac{6}{-9}$ **D** $-\left(\frac{-2}{-3}\right)$

2. Which value is greater than $-1\frac{5}{6}$?

A $-1.\overline{8}$ **B** $-1\frac{7}{8}$

C $-1.8\overline{3}$ **D** $-1\frac{4}{5}$

3. Which fraction is between -0.34 and -0.36?

A $-\frac{17}{50}$ **B** $-\frac{9}{25}$

C $-\frac{7}{20}$ **D** $\frac{35}{100}$

4. Which value equals $-3.78 - (-2.95)$?

A -6.73 **B** -0.83

C 0.83 **D** 6.73

5. Which expression does not equal $\frac{3}{5} \times \left(-\frac{6}{7}\right)$?

A $-\frac{3}{7} \times \frac{6}{5}$ **B** $\frac{3}{-5} \times \frac{6}{7}$

C $\frac{-3}{5} \times \left(\frac{-6}{-7}\right)$ **D** $\frac{-3}{-5} \times \frac{6}{7}$

6. Which value is the best estimate for $\sqrt{1.6}$?

A 2.6 **B** 1.3

C 0.8 **D** 0.4

7. Which rational number is a non-perfect square?

A $\frac{1}{25}$ **B** 0.16

C 0.9 **D** $\frac{121}{4}$

Complete the statements in #8 and #9.

8. A square has an area of 1.44 m^2. The perimeter of the square is ■ m.

9. On a number line, you would find $-3\frac{5}{11}$ to the ■ of -3.4545.

Short Answer

10. Explain why any integer is a rational number.

11. Arrange the following in descending order.

$-1.\overline{2}$ -1.2 $\frac{19}{20}$ $\frac{9}{10}$ $\frac{9}{-10}$ 0.94

12. Identify the fractions in lowest terms that are between -2 and -3 and that have 6 as the denominator.

13. Calculate.

a) $1\frac{4}{5} - 2\frac{2}{3}$ **b)** $-3.21 + 1.84$

c) $\frac{5}{8} \div \left(-\frac{11}{12}\right)$ **d)** $-2\frac{5}{7}\left(-3\frac{1}{2}\right)$

e) $-3.66 \div (-1.5)$ **f)** $-\frac{5}{6} + \left(-\frac{1}{12}\right)$

14. Canada's Donovan Bailey won the gold medal in the 100-m sprint at the Summer Olympics in Atlanta in a time of 9.84 s. He beat the second-place finisher, Frankie Fredericks of Namibia, by $\frac{5}{100}$ of a second. What was Fredericks's time?

15. What is the average of a rational number and its opposite? Explain using examples in decimal or fraction form.

16. Is 31.36 a perfect square? Explain how you know.

17. Determine.

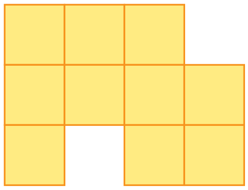
a) the number with a square root of 6.1

b) $\sqrt{0.1369}$

c) $\sqrt{7}$, to the nearest hundredth

Extended Response

18. This shape is made from ten congruent squares.



a) If the perimeter of the shape is 40 cm, what is its area?

b) If the area of the shape is 75 cm^2, what is its perimeter, to the nearest tenth of a centimetre?

19. Ron buys 75 shares in a car company. A year later, he sells the shares for $15.64 each. The result is a loss of $260.25. How much did Ron pay for each share? State any assumptions you make.

20. A Canadian quarter is made from nickel, copper, and steel. The quarter is $\frac{11}{500}$ nickel, $\frac{19}{500}$ copper, and $\frac{47}{50}$ steel.

a) Predict the sum of the three fractions. Justify your prediction.

b) Test your prediction by calculating the sum of the three fractions.

c) How many times as great is the mass of the steel as the combined mass of the nickel and the copper?

d) The mass of a Canadian quarter is 4.4 g. In a roll of 40 quarters, how much greater is the mass of copper than the mass of nickel?

Math Link: Wrap It Up!

Design a game that can be played with a partner or in a small group. The game must include

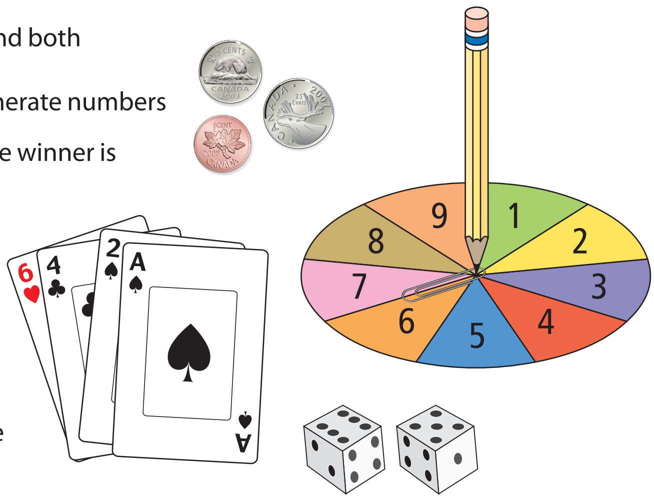
- calculations that involve at least two operations and both positive and negative rational numbers
- dice, coins, playing cards, or other materials to generate numbers

a) Describe the rules of the game, including how the winner is decided.

b) Give examples of the calculations that the game involves.

c) Play the game with a partner or in a small group.

d) Suggest alternative rules for the game. For example, you might suggest modifications to the game, such as including different operations.



Challenges

Reaction Time

An important skill drivers must have is the ability to react to obstacles that may suddenly appear in their path. You be the driver! What types of obstacles might you encounter? How quickly do you think you could react to an obstacle in the road?

You are going to calculate your reaction time.

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Materials
- 30-cm ruler

1. Work with a partner to do the following experiment.
- Your partner will hold a 30-cm ruler vertically in front of you, with the zero mark at the bottom.
- Position your thumb and index finger on each side of the ruler so that the zero mark can be seen just above your thumb. Neither your thumb nor your finger should touch the ruler.
- Your partner will drop the ruler without warning. Catch the ruler as quickly as you can by closing your thumb and finger.
- Read the measurement above your thumb to the nearest tenth of a centimetre. This is your reaction distance.
- Perform this procedure five more times, recording each distance.
- Switch roles to determine your partner's five reaction distances.

Calculate your average reaction distance.

2. The formula $d = \frac{1}{2}gt^2$ can be used to calculate reaction time, where
- d is the reaction distance, in metres
- g is the acceleration due to gravity, which is 9.8 m/s^2
- t is time, in seconds

Calculate your average reaction time. Show your reasoning.

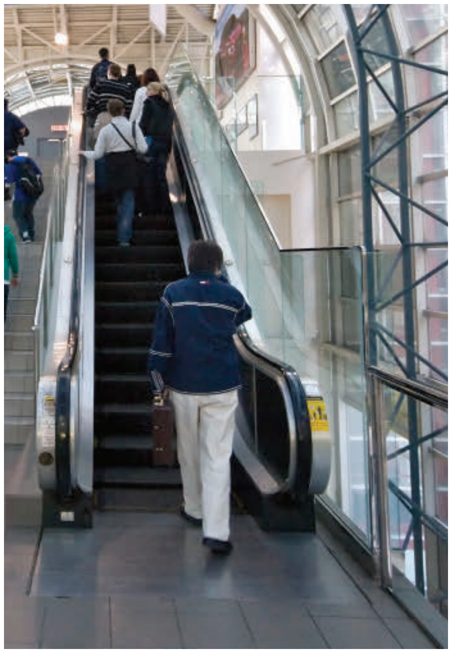
3. a) Imagine you are driving a car in a residential area and a ball rolls onto the road in front of you. You move your foot toward the brake. Based on the reaction time you calculated in #2, if you are driving at 40 km/h, how far will the car travel before you step down on the brake?
   b) What distance would you have travelled before stepping down on the brake if your original speed was 100 km/h?
   c) What other factors might influence your reaction time and your stopping distance? Share your ideas with your classmates.

Going Up?

You be an engineer! Your job is to design an elevator. It will work alongside an existing escalator to move people between levels at a local sports and entertainment arena.

1. The escalator that is already in place can move 30 people per minute from the main level to the upper level. Based on this information, how long would it take to use the escalator to move 100 people from the main level to the upper level?

2. The design of an elevator is based on the available building space, the load capacity, and the need to allow for people's personal space. An average person represents a load of 67 kg, and needs a radius of 26 cm for personal space. The area of the floor space for the elevator you are designing will be 3.75 m^2.
   a) To maximize the number of people that can be carried, what dimensions would you recommend for the elevator?
   b) Why did you choose these dimensions?
   c) What is the maximum number of people that your elevator could carry?
   d) What would be its load capacity? Justify your answer.



3. a) If the average time the elevator takes to move between the main level and the upper level is 8 s, how many people could move from the main level to the upper level in 1 min?
   b) What assumptions did you make?

4. There is a concert tonight. You want to minimize the time for moving 2000 concert goers from the main level to the upper level.
   a) What recommendations would you have for the staff who are directing people to the escalator and elevator? Justify your thinking.
   b) What assumptions did you make?